intuition. Finally, the quadratic $\mathcal{D}$-$\text{st}$ability of an antiwindup closed loop system was analyzed as an application example.

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Static Output Feedback Control Synthesis for Linear Systems With Time-Invariant Parametric Uncertainties

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Abstract—This technical note is concerned with the problem of designing robust static output feedback controllers for linear discrete and continuous-time systems with time-invariant polytopic uncertainties. Sufficient conditions for static output feedback stabilizing controller designs are given in terms of solutions to a set of linear matrix inequalities. Furthermore, the results are also extended to $H_\infty$ static output feedback controller designs. Numerical examples are given to illustrate the effectiveness of the proposed design methods.

Index Terms—Linear matrix inequalities (LMIs), parameter dependent Lyapunov functions, robust control, static output feedback (SOF), time-invariant uncertainty.

I. INTRODUCTION

Static output feedback (SOF) control is one of the most important open problems in control theory and practice. It represents the simplest closed-loop control system, which can be easily realized in practice. Therefore, the problem has been extensively studied in the past decades and for the SOF control problem of linear systems, there are various approaches to deal with it, see the survey paper [1] and the references therein. Among these existing methods, we distinguish them based on Riccati equations [2]–[4], convex approaches based on optimization techniques [5] (condition in term of min-max algorithm), [6], [7] (convex programming procedures). Because the SOF problem is nonconvex, iterative linear matrix inequality (ILMI) approaches are exploited after it is expressed as a bilinear matrix inequality formulation, see [8]–[12]. Recently, many efforts have been done to obtain sufficient linear matrix inequality (LMI) conditions for SOF controller design. Although only sufficient, the solutions have the advantage of being linear and, hence, easily tractable by standard optimization techniques. As a result, some significant advances have been achieved, see [13]–[21], and the references therein.

Many practical systems are commonly required to have good robustness so that robust control problems become important research topics in control theory. Recently, [22]–[30] have studied various robust control problems for linear discrete or continuous-time systems with uncertainties. Based on parameter dependent Lyapunov functions, [31] and [32] provide a simple and easy-to-test robust stability condition for discrete- and continuous-time uncertain linear systems with polytopic uncertainties, respectively. [33] provides robust stability evaluation for

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II. PROBLEM STATEMENT AND PRELIMINARIES

Consider a linear time-invariant system (1) with polytopic uncertainties described by state-space equations:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\
\dot{z}(t) &= C_1x(t) + D_{12}u(t) \\
y(t) &= C_2x(t)
\end{align*}
\]

(1)

where the symbol \(\delta[\cdot]\) represents the derived operator for continuous-time and the forward operator for discrete-time system. \(x(t) \in \mathbb{R}^n\) is the state vector, \(u(t) \in \mathbb{R}^r\) is the control input, \(w(t) \in \mathbb{R}^m\) is the disturbance, \(y(t) \in \mathbb{R}^p\) is the measured output, and \(z(t) \in \mathbb{R}^q\) is the controlled output. The matrices \([A]_{n \times n}, [B_1]_{n \times m}, [B_2]_{n \times p}, [C_1]_{p \times n}, [C_2]_{q \times n}, [D_{12}]_{p \times p}\) belong to the following uncertainty polytope:

\[
\Omega = \{[A]_{n \times n}, [B_1]_{n \times m}, [B_2]_{n \times p}, [C_1]_{p \times n}, [C_2]_{q \times n}, [D_{12}]_{p \times p}\} \\
\cup \sum_{i=1}^{N} \theta_i[A]_{n \times n}, [B_1]_{n \times m}, [B_2]_{n \times p}, [C_1]_{p \times n}, [C_2]_{q \times n}, [D_{12}]_{p \times p}\) \\
\times [D_{12}]_{p \times p}), \theta_i \geq 0, \sum_{i=1}^{N} \theta_i = 1\}.
\]

(2)

Assume that \(C_{2i}, 1 \leq i \leq N\), are of full row rank, and let invertible matrices \(T_i, 1 \leq i \leq N\), such that

\[
C_{2i}T_i = [I \ 0], \quad \text{for } 1 \leq i \leq N.
\]

(3)

Remark 1: For each \(C_{2i}\), the corresponding \(T_i\) generally is not unique. A special \(T_i\) can be obtained by

\[
T_i = \begin{bmatrix} C_{2i}^{-1}(C_{2i}T_i) & C_{2i}^{-1} \end{bmatrix}
\]

(4)

where \(C_{2i}^{-1}\) denotes an orthogonal basis for the null space of \(C_{2i}\).

Our aim is to design a robust output feedback controller

\[
\begin{align*}
\dot{u}(k) &= K_y(k), \quad \text{for discrete time case} \quad (5a) \\
\dot{u}(t) &= K_y(t), \quad \text{for continuous time case} \quad (5b)
\end{align*}
\]

such that the resulting following closed-loop system (6a) or (6b) is robustly stable or simultaneously meets \(H_2\) performance bound requirement.

\[
\begin{align*}
x(k + 1) &= (A + B_2K_{C_2})x(k) + B_1w(k) \\
\dot{z}(k) &= (C_1 + D_{12}K_{C_2})x(k) \\
\dot{x}(t) &= (A + B_2K_{C_2})x(t) + B_1w(t) \\
\dot{z}(t) &= (C_1 + D_{12}K_{C_2})x(t).
\end{align*}
\]

(6a) (6b)

The following preliminary lemmas will be used in this sequel.

Lemma 1: [34] If there exist a symmetric matrix \(Q\) and a matrix \(S\) such that

\[
\begin{bmatrix} Q - S - S^T & * \\
(A + B_2K_{C_2})S & -Q \end{bmatrix} < 0
\]

then the discrete-time system (6a) with \(w(k) = 0\) is asymptotically stable.

Lemma 2: [35] If there exist a symmetric matrix \(X\) and a matrix \(V\) such that

\[
\begin{bmatrix} -V - V^T & * * \\
(A + B_2K_{C_2})V + X & -X & * \\
V & 0 & -X \end{bmatrix} < 0
\]

then the continuous-time system (6b) with \(w(t) = 0\) is asymptotically stable.

Lemma 3: [37] Assume that the system (1) is with \(C_{2i} = C_i, i = 1, \ldots, N\), and \(T\) is an invertible matrix such that \(CT = [I_0]\). If there exist symmetric matrices \(Q_i, 1 \leq i \leq N\) and matrices

\[
S = \begin{bmatrix} S_{11} & 0 \\
S_{21} & S_{22} \end{bmatrix}, \quad L = [L_1 \ 0]
\]

such that

\[
\begin{bmatrix} Q_i - S - S^T & * \\
T^{-1}A_iTS + T^{-1}B_{2i}L & -Q_i \end{bmatrix} < 0, \quad \text{for } 1 \leq i \leq N
\]
then the discrete-time system (6a) is robustly stable. 

Remark 2: Lemma 3 is a direct consequence of the results in [37].

Lemma 4: [37] If there exist symmetric matrices \( \mathcal{Q}, \mathcal{Z} \) and a matrix \( \mathcal{S} \) satisfying the following LMIs:

\[
\begin{bmatrix}
\mathcal{Q} - \mathcal{S} - \mathcal{S}^T \\
(A + B_2 K C_2) \mathcal{S} - \mathcal{Q} \\
(C_1 + D_{12} C_2) \mathcal{S} - \mathcal{Q} \\
-\mathcal{Z} \\
B_1 - \mathcal{Q}
\end{bmatrix} < 0,
\]

then the continuous-time system (6b) is robustly stable with \( H_2 \) norm less than \( \sqrt{\gamma} \). 

Lemma 5: [35] If there exist symmetric matrices \( \mathcal{V}, \mathcal{Z} \) and a matrix \( \mathcal{V} \) satisfying the following LMIs:

\[
\begin{bmatrix}
\mathcal{V} - \mathcal{V}^T \\
(A + B_2 K C_2) \mathcal{V} - \mathcal{V} \\
(C_1 + D_{12} C_2) \mathcal{V} - \mathcal{V} \\
-\mathcal{Z} \\
B_1 - \mathcal{V}
\end{bmatrix} < 0,
\]

then the discrete-time system (6a) is robustly stable with \( H_2 \) norm less than \( \sqrt{\gamma} \).

Remark 3: Lemma 4 and 5 can be obtained from [37, Theorem 1] and [35, Theorem 3.3], respectively. The proofs are omitted here.

III. MAIN RESULTS

In this section, first, two LMI-based sufficient conditions for designing robust static output feedback controller for discrete and continuous-time systems are given, respectively. Then, the results are extended to robust \( H_2 \) control.

A. Robust Static Output Feedback Controller Design

The following theorem presents a robust static output feedback controller design for discrete-time systems.

Theorem 1: (Discrete-time case) If there exist symmetric matrices \( Q_i, J_{ij} \) and matrices \( S_i, 1 \leq i \leq N, L \) with

\[
S_i = \begin{bmatrix}
S_{i11} & 0 \\
S_{i21} & S_{i22}
\end{bmatrix},
\]

\[
L = \begin{bmatrix}
L_1 & 0
\end{bmatrix},
\]

\[
J_{ij} = \begin{bmatrix}
J_{i1j}^{11} & J_{i1j}^{12} \\
J_{i2j}^{21} & J_{i2j}^{22}
\end{bmatrix}
\]

satisfying the following LMIs:

\[
\begin{bmatrix}
-T_i S_i - S_i^T T_i^T \\
A_i T_i S_i + B_2 J_i L - Q_i + J_{ij}^{11} \\
J_{i2j}^{21} \\
-\mathcal{Z}
\end{bmatrix} < 0,
\]

where \( T_i, i = 1, \ldots, N \) satisfying (3) and are nonsingular, then (1) with \( w(k) = 0 \)

\[
K = L_1 S_{i11}^{-1}
\]

is robustly stable.

Proof: From the structure of \( L, S_i \) and (3), (10), we can obtain

\[
L = \begin{bmatrix}
K S_{i11} & 0
\end{bmatrix} = \begin{bmatrix}
K & 0
\end{bmatrix} \begin{bmatrix}
S_{i11} & 0 \\
S_{i21} & S_{i22}
\end{bmatrix}
\]

\[
= K \begin{bmatrix}
I & 0
\end{bmatrix}^{-1} T_i S_i = KC_i \begin{bmatrix}
T_i, S_i
\end{bmatrix}.
\]

Substituting \( L \) for \( KC_2, T_i, S_i \) in (8), then (8) can be rewritten as follows:

\[
\begin{bmatrix}
-\mathcal{Z} \\
A_i T_i S_i + B_2 J_i C_2i - Q_i + J_{ij}^{11} \\
J_{i2j}^{21} \\
Q_i W_i
\end{bmatrix} < 0,
\]

\[
1 \leq i, j \leq N.
\]

Multiplying (12) by \( \theta_i, \beta_i \) and summing them, then we have

\[
\begin{bmatrix}
-\mathcal{Z} \\
A_i + B_2 J_i C_2i - \mathcal{Q} \\
\mathcal{Q} W_i
\end{bmatrix} < 0
\]

where \( A, B_2, C_2 \) are same as in (2), and

\[
\begin{bmatrix}
\mathcal{V} - \mathcal{V}^T \\
A_i + B_2 J_i C_2i - \mathcal{Q} \\
\mathcal{Q} W_i
\end{bmatrix} < 0
\]

Let \( S = \mathcal{W}^{-1} \) and pre- and postmultiply (13) by \( \text{diag} \{ S^T \mathcal{II} \} \) and its transpose, then we can obtain

\[
\begin{bmatrix}
-\mathcal{S} - \mathcal{S}^T \\
(A_i + B_2 J_i C_2i) S - \mathcal{Q} \\
\mathcal{Q}
\end{bmatrix} < 0
\]

On the other hand, pre- and postmultiply (9) by \( \theta_i I \) and its transpose, it follows that:

\[
\sum_{i=1}^{N} \sum_{j=1}^{N} \theta_i \beta_j J_{ij} + \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_i \beta_j J_{ij}^{T} = 2 \mathcal{J} \geq 0.
\]

From (14) and (15), then we have

\[
\begin{bmatrix}
-\mathcal{S} - \mathcal{S}^T \\
(A_i + B_2 J_i C_2i) S - \mathcal{Q} \\
\mathcal{Q}
\end{bmatrix} < 0
\]
which is equivalent to

\[
\begin{bmatrix}
Q - S - S^T \\
(A + B_2 K C_2) S
\end{bmatrix} < 0.
\]

(16)

From (16) and Lemma 1, we can obtain that the system (1) is robustly stable. Moreover, from (1), we can deduce that the matrices \( T_i S_i \) are positive-definite (not necessarily symmetric) which implies that the matrices \( S_i \) and implicitly \( S_{i1} \), are invertible because \( T_i \) are invertible. Then \( L_1 = K S_{i1} \) admits the solution (10). Thus, the proof is complete.

By combining Theorem 1 and the technique of [32], we can have the following corollary.

**Corollary 1:** If there exist symmetric matrices \( Q_i, 1 \leq i \leq N \) and matrices \( S_i, 1 \leq i \leq N, L \) with the same structures as in (7), satisfying the following LMIs:

\[
\begin{bmatrix}
-T_i S_i - S_i^T T_i^T \\
A_i T_i S_i + B_{2i} L - Q_i
\end{bmatrix} < \begin{bmatrix}
0 & * \\
0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-T_i S_i - S_i^T T_i^T \\
A_i T_i S_i + B_{2i} L - Q_i
\end{bmatrix} < \begin{bmatrix}
\frac{1}{N-1} & * & * \\
0 & 0 & 0
\end{bmatrix}
\]

\[1 \leq i \neq j \leq N \]

where \( T_i, i = 1, \ldots, N \), satisfying (3) and are nonsingular, then (1) with \( w(k) = 0 \) and (10) is robustly stable.

**Proof:** By the technique similar to the ones used in [32] and using the method of Theorem 1, the proof is easily obtained and omitted.

**Remark 4:**

i) Theorem 1 presents new sufficient conditions for designing robust static output feedback controllers for discrete-time systems which are of LMIs and can be effectively solved via LMI Control Toolbox [38].

ii) Since \( C_{2i} \), usually are not square matrices, then the null space of \( C_{2i} \) are not zero. Therefore, the introduced slack variable \( S \) can be dependent on the uncertain parameter \( \theta_i \), which is helpful for reducing the conservatism of the new proposed design condition. Compared to the result of [37], the new result is applicable for system output matrix \( C_2(\theta) \) with uncertainties. But, the system output matrix \( C_2(\theta) \) must be fixed in [37], i.e., \( C_2(\theta) \) is independent parameter \( \theta_i \). Even if the system output matrix is not uncertain, i.e., \( C_{2i} = C_{2i}(1 = 1, \ldots, N) \), the new design condition also provides less conservative results than the existing design method based on the technique of [32] (Corollary 1), which has been shown in Example 1.

iii) By considering the relations of parameter \( \theta_i \), a type of slack variables are introduced for less conservative stability conditions for polytopic linear systems in [33]. In Theorem 1, the type of slack variables (i.e., \( J_{ij} \)) are also exploited. However, in order to obtain convex conditions, the technique is only used for one part of the global matrix inequality, i.e., the type of slack variables only emergence on the blocks (2,2), (2,3), (3,2), (3,3) of (8) and (13). If the type of slack variables in [33] is applied to the global matrix inequality, less conservative result can be given. But, the corresponding conditions will become nonlinear, and, hence, not convex.

iv) It should be noted that Corollary 1 is obtained based on the technique from [32]. If the conditions in Corollary 1 hold, then choose

\[
J_{ij} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, J_{ij} = -\frac{1}{N-1} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}
\]

furthermore

\[
[J_{ij}]_{N \times N} + [J_{ij}]^T_{N \times N} 
\]

\[
= 2 \begin{bmatrix}
\frac{1}{N-1} I & \cdots & \frac{1}{N-1} I \\
\vdots & \ddots & \vdots \\
\frac{1}{N-1} I & \cdots & \frac{1}{N-1} I
\end{bmatrix} \geq 0
\]

therefore, the conditions of Theorem 1 hold, which shows that the conditions of Theorem 1 are less conservative than those of Corollary 1.

It should be noted that for each \( C_{2i} \), there may exist different choices of \( T_i \) satisfying (3). The following theorem shows that the feasibility of the condition of Theorem 1 is independent of the choices of \( T_i \).

**Theorem 2:** If the conditions of Theorem 1 are feasible for some \( T_i \) satisfying (3), then they are feasible for any \( T_i \) satisfying (3), i.e., \( C_{2i} = C_{2i}[0] \).

**Proof:** Since \( T_i \) and \( \tilde{T}_i \) satisfy (3), \( C_{2i} \tilde{T}_i = C_i \tilde{T}_i = I \), which implies that \( [I \ 0] \tilde{T}_i^{-1} = [I \ 0] \tilde{T}_i^{-1} \). Postmultiplying it by \( T_i \), then we have

\[
[I \ 0] = [I \ 0] \tilde{T}_i^{-1} T_i.
\]

Denote

\[
H_{i} = \tilde{T}_i^{-1} T_i = \begin{bmatrix} H_{i1}^{12} \\ H_{i2}^{12} \end{bmatrix}
\]

then from (17), it follows that: \( H_{i1}^{i} = I, H_{i2} = 0 \). Consider

\[
T_i S_i = \tilde{T}_i H_{i} S_i = \begin{bmatrix} I & 0 \\ H_{i1}^{21} & H_{i2}^{22} \end{bmatrix} \begin{bmatrix} S_{i1} \\ S_{i1}^{21} \end{bmatrix} = \begin{bmatrix} S_{i1} \\ \tilde{S}_{i2} \end{bmatrix}
\]

\[(18)
\]

where \( \tilde{S}_{i2} = H_{i2}^{12} S_{i1} + H_{i2}^{22} S_{i2}, \tilde{S}_{i2} = H_{i2} S_{i2} \). Let

\[
\tilde{S}_{i1} = \begin{bmatrix} S_{i1} \\ \tilde{S}_{i2} \end{bmatrix}
\]

then (18) can be rewritten as \( T_i S_i = \tilde{T}_i \tilde{S}_i \). Therefore, if (8) holds for \( T_i \), then (8) holds for \( \tilde{T}_i \), which implies that the conditions of Theorem 1 is feasible for \( \tilde{T}_i \). Thus, the proof is complete.

For continuous-time case, we have Theorem 3.

**Theorem 3. (Continuous-Time Case):** If there exist symmetric matrices \( X, J_{ij}, 1 \leq i, j \leq N \) and matrices \( V, 1 \leq i \leq N, L \) with

\[
V = \begin{bmatrix} V_{11} & 0 \\ V_{21} & V_{22} \end{bmatrix}, L = \begin{bmatrix} L_1 & 0 \end{bmatrix},
\]

\[
J_{ij} = \begin{bmatrix} J_{ij}^{11} & 0 \\ J_{ij}^{12} & J_{ij}^{22} \end{bmatrix}
\]
satisfying the following LMI s:

\[
\begin{bmatrix}
-T_i V_i - V_i^T T_i^T \\
A_i T_i V_i + B_{2j} K C_{2j} T_i V_i + X_j \\
T_i V_i
\end{bmatrix} < 0, \quad 1 \leq i, j \leq N.
\]

From Lemma 2, it follows that (1) is robustly stable. Moreover, from (19), we can deduce that the matrices \( T_i V_i \) are positive-definite (not necessarily symmetric) which implies that the matrices \( V_i \), and implicitly \( V_{11} \), are invertible because \( T_i \) are invertible. Then \( L_1 = KV_{11} \) admits the solution (21). Thus, the proof is complete.

**Remark 5:** Similar to Theorem 2, it can be shown that the feasibility of the condition of Theorem 3 is also independent of the choices of \( T_i \).

### IV. Example

In this section, three numerical examples are presented to illustrate the effectiveness of the proposed methods. In Example 1, a comparison between Theorem 1, Corollary 1 and Lemma 3 is given. The design
methods given by Theorems 4 and 5 are illustrated by Examples 2 and 3, respectively.

Example 1: Consider a discrete-time system which belongs to the 2-polytopic convex polyhedron in the form of (2) with \( w(k) = 0 \) and

\[
A_1 = \begin{bmatrix} 1.2 & 2 \\ 1 & -0.7 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.1 & 1 \\ 0 & -0.6 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 \\ 1.1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

For this example with the fixed output matrix, Lemma 3, Theorem 1, and Corollary 1 are applicable for designing robust static output feedback controllers. The LMIs of Lemma 3 are infeasible, and the ones of Corollary 1 are also infeasible. However, by Theorem 1, a stabilizing controller gain can be obtained as \( K = -0.7589 \). Therefore, the method given by Theorem 1 provides a better alternative design for this example.

Example 2: Consider a discrete-time system which belongs to the 2-polytopic convex polyhedron in the form of (2) with

\[
A_1 = \begin{bmatrix} 1.6 & 0 \\ 1 & 0.7 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.1 & -0.3 \\ 0 & 0.1 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}, \quad C_{11} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

\[
D_{121} = 1, \quad D_{122} = 0.9
\]

By (4), we can obtain

\[
T_1 = \begin{bmatrix} 0.9901 & 0.0995 \\ -0.0990 & 0.9950 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.9901 & -0.0995 \\ 0.0990 & 0.9950 \end{bmatrix}
\]

satisfying (3).

By applying Theorem 4 with and without the slack variables \( J_{ij} \), the guaranteed \( H_2 \) performance \( \sqrt{\gamma} \) and the corresponding static output feedback gain \( K \) are given in Table I, respectively. The computation results show that the design method given by Theorem 4 is less conservative than the one given by Theorem 4 without the slack variables \( J_{ij} \) (i.e., \( J_{ij} = 0 \)).

\[
\begin{array}{|c|c|c|}
\hline
\text{Theorem 4 without } J_{ij} & \text{Theorem 4 with } J_{ij} \\
\hline
\sqrt{\gamma} & 0.5656 & 0.5512 \\
K & -0.8583 & -0.8428 \\
\hline
\end{array}
\]

Example 3: Consider a continuous-time system which belongs to the 2-polytopic convex polyhedron in the form of (2) with

\[
A_1 = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & -1 \\ 0 & -5 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_{11} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad D_{121} = 1, \quad D_{122} = 2.
\]

By (4), we can obtain

\[
T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.4000 & -0.4472 \\ 0.2000 & 0.8944 \end{bmatrix}
\]

satisfying (3).

The guaranteed \( H_2 \) controllers are obtained by Theorem 5 with and without the slack variables \( J_{ij} \), respectively, which are given in Table II. The computation results also show that the design with the slack variables \( J_{ij} \) can reduce conservatism.

\[
\begin{array}{|c|c|c|}
\hline
\text{Theorem 5 without } J_{ij} & \text{Theorem 5 with } J_{ij} \\
\hline
\sqrt{\gamma} & 14.8348 & 8.0343 \\
K & -2.7524 & -2.9325 \\
\hline
\end{array}
\]

V. Conclusion

In this technical note, the problem of designing robust SOF controllers for linear discrete and continuous-time systems with time-invariant polytopic uncertainties has been investigated. LMI-based sufficient conditions for SOF stabilizing controller design are given, and the results are also extended to \( H_2 \) SOF controller design. Compared to existing results, the new proposed approach is applicable for linear
system with time-invariant polytopic uncertainties, which may simultaneously emerge on system output and input matrices. In the technical development, the properties of the null space of system output matrices are exploited and a parameter-dependent slack variable is introduced to separate system matrix and Lyapunov matrix, which are helpful for reducing the conservatism of the obtained conditions for designing SOF controllers. The numerical examples have shown the effectiveness of the proposed design methods.

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