Some Nonlinear Parameters of HRV Signals for Healthy and Arrhythmia Human

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Abstract: The HRV (Heart Rate Variability) signals of healthy people or not have different nonlinear characteristics. In the paper, the extraction of HRV signals from ECG is achieved by picking up R peaks of ECG with wavelet transform technique. Some nonlinear parameters (correlative dimensions, maximum Lyapunov exponents, complexity and approximate entropy) of HRV signals of both healthy and arrhythmia people are calculated. These nonlinear parameters of HRV signals are helpful in analyzing human state and diagnosing of heart diseases.

Keywords: HRV, Phase space reconstruction, Correlative dimension, Lyapunov exponent, Complexity, Approximate entropy

1. Introduction

Heart rate variability (HRV) is obtained from the measurement of the RR intervals by transforming the electrocardiogram (ECG) signal into an event series, the events being the QRS reference moments that are obtained with the help of a QRS detector [1]. It is known that HRV is mainly caused by cardiovascular neuro-autonomic control. HRV includes so much information about the heart blood vessel neural system and body fluid regulation that it can be used to diagnose, treat or predict heart and blood concerned diseases. Many researchers have already been benefited from the HRV analyses.

Nowadays, some linear analyses, including statistical, spectral and transfer function method are used for HRV [2]. RR intervals are compared qualitatively with their mean values mRRs and standard covariance SDRR, or with their power spectral density spectrums in different frequency ranges. Meanwhile, some nonlinear analyses, such as Poincare mapping, fractal dimension and complexity, are also adopted to analyze to different disease subjects with HRV.

However, most of these methods cannot give a quantitative description of HRV, especially when its nonlinearity is considered. In fact, it is already suggested that complex fluctuations in healthy heart rate may reflect a deterministic chaos. The recent research shows that a perturbation of heart activity may lead to a loss of complexity of the heart rate dynamics. In this way, the nonlinearity of HRV could be used to detect abnormal heart states.

In this paper, some nonlinear characteristic parameters of HRV in two typical states, healthy adult (normal heart rate) and unhealthy (arrhythmia), are calculated comparatively, with the main diagnosis purpose of heart diseases. In Section 2, an extracting algorithm of HRV from ECG is discussed. In some other research works, the band-pass filtering is adopted to get R wave, in which the baseline excursion and de-noise must be enforced exactly [3]. The wavelet transform technique is adopted here to ascertain R peaks of ECG. A cubic spline function is used to interpolate the obtained RR interval event series, and HRV time series are attained via re-sampling in a proper frequency. In Section 3, the nonlinear theory is applied to calculate some nonlinear parameters of HRV time series data of normal and arrhythmia states, including the correlative dimensions of reconstructed phase space, the largest Lyapunov exponents, approximate entropy, and complexity. These calculated nonlinear parameters of HRV are significant for understanding the nonlinear characteristics of ECG and essential for quantitative diagnosing of some heart diseases.

2. Extraction of HRV time series

The average voltage range of ECG signals is 0.05–5mV, and about 0.05-100Hz (3dB) in frequency domain. ECG is often contaminated by muscle electricity pulse and other noises.

In order to detect R peaks accurately, a proper mother wavelet function is chosen and it should be symmetric. A symmetrical wavelet converts the peaks of signal into maximum values, whereas a nonsymmetrical one will convert signal peaks into zeros. A mother wavelet of square spline is adopted here and a 3-level wavelet transform is achieved, which is shown in Fig. 1.

The decomposition results of ECG are shown in Fig. 2. It can be seen that d1, d2 and d3 are the high-frequency detailed components that contain most energy of ECG, whereas a3 represents the low-frequency approximate component in which there is little energy. The detection of R peaks is based on the high frequency components.

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The R peaks are then extracted by using a proper threshold, since a too big threshold would cause peak losing and a too small one would get false peaks. In this paper, we adopt the average value of high frequency signal...
as threshold 1 and its half value as threshold 2. When scanning high frequency signal, the signal values exceeding threshold 1 are marked as position 1 and marked as position 2 if less than threshold 2. There will exist a R peak between position 1 and 2.

![Signal decomposition diagram](image)

**Fig. 1** A 3-level wavelet decomposition

![ECG signal decomposition](image)

**Fig. 2** Wavelet decomposition results for an ECG signal

Due to the uneven sampling frequency of the R peak series, it cannot be analyzed as HRV signals. Here the R peak series is interpolated and re-sampled. The interpolation is carried out with a cubic spline interpolation, but not a linear one with two data points, all over the whole R peak series. Based on the interpolated signal, re-sampling under a proper sampling frequency is applied.

Two groups of measured ECG signals and positions of R peaks are shown in Fig. 3(a, b). The corresponding two HRV time series are shown in Fig. 4(a, b), where the horizontal axis is re-sampling points and the vertical axis is RR intervals in second. The re-sampling frequency is 16.67Hz.

![ECG and HRV plots](image)

**Fig. 3** Checked R peaks and original ECG signals

**Fig. 4** Two set of typical HRV time series
3. Nonlinear parameters of HRV

3.1 Correlative dimensions

The fractal characteristics of HRV could be quantitatively described by correlative dimension [4,5]. Firstly, to perform the time series in a phase space with a state vector with an embedding coordinates. An optimum delay time is selected via calculating the first minimum of the auto mutual information function, to ensure that the reconstructed phase space can represent the natural characteristics of original time series. The optimum minimal embedding dimension is ascertained according to reference [6]. Correlative dimension of reconstructed HRV data is calculated based on the optimum delay time and minimal embedding dimension.

For an observed time series \( x(n) \) of HRV, the reconstructed phase space and its vectors are

\[
X = \{X_i\}, \quad X_i = (x_i, x_{i+\tau}, \ldots, x_{i+(m-1)\tau})
\]

where, \( \tau = k\Delta t \), is delay time, \( k \) is positive integer, \( \Delta t \) is the sampling interval of time series. \( i = 1, 2, \ldots, N_m \), \( N_m = N - (m - 1)\tau \), \( m \) is the embedding dimension.

The delay time \( \tau \) is estimated via calculating first minimum of the auto mutual information function that is an index of the general stochastic correlation of two variables. The auto mutual information function is defined as

\[
I(\tau) = \sum_{i,j} P_i(\tau) \cdot \ln \frac{P_i(\tau)}{P_i P_j}
\]

where, \( P_i \) is the probability distribution of time series \( x(n) \) at each point. \( P_i \) is the probability in domain \( i \), and \( P_j(\tau) \) is the probability of the state that \( x_i \) is in domain \( i \) and \( x_{i+\tau} \) is in domain \( j \). When \( I(\tau) \) reaches its first minimum, the corresponding \( \tau \) is the rational delay time.

Considering a limited time series, correlative integration is

\[
C_m(r) = \frac{2}{N_m(N_m - 1)} \sum_{i=1}^{N} \sum_{j=1}^{N} H(r - \|x_i - x_j\|)
\]

where, \( H \) is Heaviside function, and if \( x > 0 \), \( H(x) = 1 \), if \( x \leq 0 \), \( H(x) = 1 \). The correlative integration can be rewritten in exponential form

\[
C_m(r) \propto r^{D_2}
\]

Assume

\[
d_2(r, N_m) = \frac{\partial \ln C_m(r, N_m)}{\partial \ln(r)}
\]

The correlative dimension is defined as

\[
D_2 = \lim_{{r \to 0}} \lim_{{N_m \to \infty}} d_2(r, N_m)
\]

The slope in the middle straight segment of the curve \( \ln C_m(r) \) to \( \ln(r) \) is correlative dimension \( D_2 \).

3.2 Largest Lyapunov exponents

The average radiation velocity of the adjacent orbits in phase space could be quantitatively described by the largest Lyapunov exponent, which is also one of the most important indexes to estimate whether the system is in chaos or not. There are lots of methods to calculate the largest Lyapunov exponents from a nonlinear time series [7, 8]. In this paper, phase space reconstruction of HRV is firstly achieved, and then the largest Lyapunov exponents are calculated. The result indicates that the largest Lyapunov exponents of HRV in both healthy and unhealthy state are all positive, but the value of the latter is comparatively small.

Considering two trajectories in space \( X \), \( L_1 \) and \( L_2 \), with initial points of \( x_0 \) and \( x_0 + \Delta x_0 \), the former is taken as the standard trajectory and the latter as the adjacent one. At time \( t \), the distance of the two points on the two different trajectories is \( w(x_0, t) = x(x_0 + \Delta x_0, t) - x(x_0, t) \).

When \( w \) is small, the exponential departure of the two trajectories is

\[
\lambda = \lim_{{w \to 0}} \frac{1}{t} \ln \frac{\|w\|}{\|w_0\|}
\]

where, \( w_0 = w(x_0, 0) \). In phase space with \( m \) dimensions, all the vectors \( w \) form a \( m \) dimensional moving tangent space along with the phase trajectory. Choosing a set of coordinate units of the space and rearranging them into a sequence numerically, the phase trajectory. Choosing a set of coordinate units of the space and rearranging them into a sequence numerically, the largest Lyapunov exponents are obtained as

\[
\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n
\]
Considering that the largest Lyapunov exponents obtained here are only statistical average of the exponential radiation of each point on its trajectory, as $p$ is a large finite integer.

### 3.3 Approximate entropy

Approximate entropy is an important quantitative index to describe chaos in phase space, and it reflects the appearance possibility of a new model when dimension of phase space increases $[9, 10]$. The edge probability distribution is used to distinguish different processes, including regular movement, chaos, and random movement. Moreover, approximate entropy also can be used to measure the lost rate of data information. Positive and limitary entropy indicates that the time series and its inherent dynamics are chaotic.

Approximate entropy is deduced based on the concept of correlative integration of a time series. For a time series $x(n), n=1,2,...,N$, its correlative integration is defined as Eq. 3. Take the Logarithm form of correlative integration $C_n(r)$ and note as $\Phi^n(r)$. The dimension of reconstructed phase space increases from $m$ to $m+1$, and then the logarithm correlative integration is calculated again getting $\Phi^{n+1}(r)$. When the length of time series is a finite number of $N$, the approximate entropy is estimated as

$$E_A(m,r,N) = \Phi^n(r) - \Phi^{n+1}(r)$$

(11)

Obviously, the value of approximate entropy $E_A$ is related to the values of $m$ and $r$. The initial values of $m$ and $r$ could be taken as 2 and $0.1 \sim 0.2\sigma$, respectively, where $\sigma$ is the standard deviation of the original data.

### 3.4 Complexity

Complexity reflects the appearing speed of a new model with its length in one dimensional time series. The complexity of HRV time series is proposed to describe its nonlinearity, which is calculated based on L-Z algorithm as referring $[11, 12]$. The value of complex degree of regular movement (stable and periodic) is 0, while that of random movement (white noise) is 1, and that of regular movement with noise, color noise or chaos is often between 0 and 1.

Here the complexity for a character string is taken to demonstrate the L-Z algorithm. Assume that $S$ consists of sub-strings, $S = S_1S_2\cdots S_n$, and $Q$ is a new string which is just added to $S$. $SQ$ represents the new string formed by $S$ and $Q$. $SQ\pi$ represents the string $SQ$ with the last character omitted, and $V(SQ\pi)$ represents the set of all the sub-strings of $SQ\pi$. If $Q$ is contained within $S$ (that is, $Q$ is equal to $S_i$, a sub-string of $S$), it is called a copy of $S$, and the value of complexity remains unchanged. In contrast, if $Q$ is not contained in $S$, it is called an insert, and the value of complexity will increase by 1 (where once insert occurs).

The whole complexity, $C(N)$, is obtained after all operating stated above along the total string. Then a normalized complexity can be defined as

$$C_{lz}(N) = C(N) / B(N)$$

(12)

where, $B(N) = N / \log_2 N$, $N$ is the length of total string.

### 4. Results

Analyze the two sets of typical HRV time series shown in Fig. 4. The phase space reconstruction is conducted with time-delay coordinate method at the first step. The best time delay is determined with auto mutual information method. Here, the two values of delay time for healthy and unhealthy HRV are 24 and 18, respectively. Then, the minimum embedding dimensions of the two time series are, as curve kinks, 3 and 4 in Fig. 5, respectively, just according to the calculating method in reference [6].

The curve slopes in Fig. 6(a, b) are estimated with the least square method, based on the linear segments of $\ln C_n(r)$ to $\ln(r)$, which are the logarithms of correlative integration and measuring scale. The slope values, i.e., correlative dimensions of the two sets of HRV, are 2.7561 and 3.5515, respectively. These correlative dimensions demonstrate that HRVs are fractal in geometry.
The estimated values of largest Lyapunov exponents of two sets of HRV, corresponding to the healthy and unhealthy state shown in Fig. 4, are 1.4998 and 0.8088, respectively. Both of them are positive. The two positive values of largest Lyapunov exponents imply that these two HRV time series are both chaotic. Numerically, the largest Lyapunov exponent in unhealthy state is smaller than that in healthy state, that is, the chaos in unhealthy state (arrhythmia) is weaker than that in healthy state (normal heart rhythm).

Approximate entropy values of HRV time series in the two different healthy states are calculated; the results are both limitary, while showing apparent difference. The calculated approximate entropies are shown in Fig. 7.

As shown in Fig. 8, the complexities of the two sets of HRV in both healthy and unhealthy state are illustrated, and the former turns to be greater than the latter. The calculated results here show that the complex degree of HRV in healthy state is obviously higher than that in unhealthy state.

Comparatively, fractal dimension, Lyapunov exponents, and approximate entropy could describe the nonlinearity of system in certain ways. Fractal dimension only describes the static portrait of system in phase space without concerning dynamic characteristics, on the contrary is the Lyapunov exponent. Approximate entropy fails to work for signals contaminated with loud noise. The calculated approximate entropies appear to be much similar to complexity results.

5. Conclusions

HRV is known to reflect human-body health state and mental directly, and it features apparent nonlinear characteristics. In the paper, several nonlinear characteristic parameters of HRV time series are calculated in order to study the difference between different human-body health states.

R peaks are extracted with wavelet transform from original ECG signals, and the HRV time series are obtained via re-sampling on R-R intervals.

Based on the optimized delay time and the minimal embedded dimension for phase space reconstruction, the correlative dimension is calculated and the largest Lyapunov exponents are estimated. The obtained results show that, the largest Lyapunov exponents of HRV in healthy state (normal heart rhythm) and unhealthy state (arrhythmia) are both positive, but the former one is larger than the latter. The results of approximate entropy and complexity appear to be similar, and both the approximate entropy and complexity in healthy state are much higher than those in unhealthy state.

The analysis of HRV time series in this paper is supposed to be important for further understanding of nonlinear characteristics of ECG signals and is significant in quantitative diagnosing of some heart diseases.

References


