Nonlinear Dynamics of Mechanical Systems with Sectional Frictions

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**Abstract.** This paper presents mainly the nonlinear dynamics of the mechanical system with sectional frictions and combination friction coefficients. It is clear that the nonlinear dynamic characteristics of mechanical systems with sectional frictions are quite different from those of classical machinery, and have more precise and valuable in many practical projects. The expressions of various nonlinear forces are given firstly and the approximation solutions of the system are found with asymptotic method in nonlinear theory, and the combination friction coefficients and damping coefficients of the materials are obtained. Then some nonlinear dynamic characteristics of the system with sectional frictions are also discussed. The results are very important for designers of these machines.

**Introduction**

In some machines such as the vibrating conveyer, the vibrating feeder, the vibrating cooling machines etc., materials on the working body will move in various forms. In the process of motion, the materials on the working body will involve various kinds of nonlinear force, namely, dry friction, sectional inertial force etc [1]. So far the nonlinear dynamic characteristics of mechanical systems with sectional friction have not been fully understood [2, 3].

In order to analyze the vibrating systems with the nonlinear forces, and investigate the influence of the material on motion of a vibrating body, the expressions of various nonlinear forces must be given preliminarily. By using the successive approximation method in nonlinear theory, the approximation solution of the system will be found [4–6].

Based on the above, we may present the calculation method of the combination friction coefficient and damping coefficient of the materials on working body with vibration. The above data are very important for designing these machines. In addition, we will also discuss the nonlinear dynamic characteristics of the systems of these machines with sectional friction force and inertial force. Besides theoretical research, we have carried out some experiments with sectional friction and inertial forces. The experiments will verify the above theoretical results.

**The Model of Nonlinear Vibration of the Vibrating Body with Sliding Material**

Figure 1 shows the mechanical model of an ore cooling machine supported by elastic connecting rods. When the motion acceleration is small, the materials on it will slip only and always keep contact with the vibrating body in the vertical direction \( y \). Hence the nonlinear equation of motion of the vibrating body can be expressed as follows:

\[
m_p \dddot{s} + f_s \ddot{s} + m_m \dot{s} \dot{s} + m_m \dot{s} \dddot{s} \sin^2 \dot{\delta} + eF_m (\ddot{x}, \dot{x}, x) \cos \delta + k_s \sin \nu t - s = k_o (\nu t - s).
\]

where, \( m_p \) is the mass of the vibrating body; \( m_m \) is the mass of materials; \( f_s \) is the damping coefficient; \( k \) is the spring stiffness; \( k_o \) is the elastic connecting rod stiffness; \( s, \dot{s}, \ddot{s} \) are the displacement, velocity and acceleration of the body in the vibration direction; \( \dot{\delta} \) is the vibrating direction angle.
As shown in Fig. 2, the nonlinear force $\varepsilon F_m(\ddot{x}, \dot{x}, x)$ of the materials acting on the vibrating body can be written as

$$
\varepsilon F_m(\ddot{x}, \dot{x}, x) = \begin{cases} 
    m_\text{m} \ddot{s} \cos \delta_0 & \text{when } \Phi_e - 2\pi \leq \Phi \leq \Phi_k, \Phi_m \leq \Phi \leq \Phi_q \\
    -m_{\text{mf}} \ddot{f} g + \dddot{s} \sin \delta & \text{when } \Phi_k \leq \Phi \leq \Phi_m \\
    m_\text{m} \ddot{f} g + \dddot{s} \sin \delta & \text{when } \Phi_q \leq \Phi \leq \Phi_e 
\end{cases}.
$$

(2)

Where, $f$ is the sliding friction coefficient between materials and vibrating body, $\Phi_k$ (or $\Phi_q$) and $\Phi_m$ (or $\Phi_e$) are the phase angles of the beginning and ending of the material slips in positive and negative directions.

The Solution of Nonlinear Equation of the Vibrating Body with Sliding Materials

The vibrating machine works near resonance point generally, hence the approximation solution will be found in the case of main resonance. The first approximation solution may be shown as:

$$
s = a \cos(vt + \theta) = a \cos \psi.
$$

(3)

$$
\dot{s} = -a v \sin(vt + \theta).
$$

(4)

$$
\ddot{s} = -a v^2 \cos(vt + \theta).
$$

(5)

The nonlinear force of regional friction can be written as

$$
f_\alpha(a, \psi) = \varepsilon F_m(-a v^2 \cos \psi, -a v \sin \psi, \alpha \sin \psi) \cos \delta.
$$

(6)

According to the asymptotic method (KBM) in nonlinear analytical theory, the combination stiffness coefficient and damping coefficient can be found and listed as follows:

$$
K_m = \frac{m_\text{m} v^2 a \sin^2 \delta - b_c \cos \delta}{m_\text{m} v^2 a} = \sin^2 \delta - \frac{b_c \cos \delta}{m_\text{m} v^2 a}.
$$

(7)
\[ f_m = \frac{a_i \cos \delta}{v_a} \]  

Where,

\[ a_i = \frac{m_m v^2 a}{\pi} \left\{ \frac{1}{2} \cos \delta \sin^2 \Phi \begin{bmatrix} \Phi_k & 1 - \cos \delta \sin^2 \Phi \\ \Phi_{e-2\pi} & 1 - \cos \delta \sin^2 \Phi \end{bmatrix} \right\} 
\]

\[ b_i = \frac{m_m v^2 a}{\pi} \left\{ \frac{1}{2} \cos \Phi \begin{bmatrix} \Phi_k & 1 - \sin \Phi \sin \delta \end{bmatrix} \right\} 
\]

\[ f = \frac{g}{v^2 a} \left\{ \begin{bmatrix} \Phi_m + f \left[ \frac{g}{v^2 a} \sin \Phi \sin \delta \end{bmatrix} \right] \right\} 
\]

The approximation solutions of first order of amplitude and phase difference of angle are:

\[ a = \frac{k_a \sin \alpha}{k + k_b \left\{ m_p + \left( \sin^2 \delta \right) \frac{b \cos \delta}{m_m v^2} \right\} v^2} \]  

\[ \alpha = \arctan \frac{f_s + \frac{a_l \cos \delta}{v_a}}{k + k_b \left\{ m_p + \left( \sin^2 \delta \right) \frac{b \cos \delta}{m_m v^2} \right\} v^2} \]  

Only when the material slips, it can be presented that the combination coefficient \( K_m < 1 \) and the damping coefficient \( f_m > 0 \).

An example is used to prove the above theoretical results. The vibratory cooling machine is excited in \( n = 330 \text{r/min} \), its vibration amplitude is \( \lambda = 14.5 \text{mm} \) and vibration angle is \( \delta = 22^\circ \). The friction coefficient between materials and body is \( f_b = 0.95 \). The slope angle is \( \mu = 43^\circ 40' \). Then the regional angles of friction can be calculated as \( \Phi_k = 25^\circ, \Phi_m = 227^\circ, \Phi_q = 252^\circ, \Phi_v = 305^\circ \). The obtained results are

\[ a_i = \frac{m_m v^2 a}{\pi} (0.229 - 0.17 + 0.69 + 0.116) = 0.272 m_m v^2 a \]  

\[ b_i = \frac{m_m v^2 a}{\pi} (-0.25 - 0.3 - 0.25 - 0.776) = -0.5 m_m v^2 a \]  

\[ K_m = \sin^2 \delta \frac{b \cos \delta}{m_m v^2} = \sin^2 22^\circ + 0.5 \cos 22^\circ = 0.375^2 + 0.5 \times 0.93 = 0.604 \]  

\[ f_m = \frac{a_i \cos \delta}{v_a} \frac{0.272 m_m v^2 a \cos 22^\circ}{v_a} = 0.2520 m_m v \]  

Some other calculated results with above equations are list in Table 1.

In the cases of amplitudes \( a = 13, 14.5, 16, 17.5 \text{ mm} \), the combination coefficient of materials changes from 0.4–0.7, and the damping coefficient of materials vary from 0.25 \( m_m \) to 0.33 \( m_m \).
Table 1 the combination stiffness coefficients and damping coefficients in different vibration cases

<table>
<thead>
<tr>
<th>λ[mm]</th>
<th>Φₜ</th>
<th>Φₙ</th>
<th>Φ₂</th>
<th>Φ₃</th>
<th>b₁</th>
<th>Kₙ</th>
<th>a₁</th>
<th>fₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>28°</td>
<td>224°</td>
<td></td>
<td></td>
<td>−0.5mₙv²a</td>
<td>0.66</td>
<td>0.27mₙv²a</td>
<td>0.25mₙv</td>
</tr>
<tr>
<td>14.5</td>
<td>25°</td>
<td>227°</td>
<td>252°</td>
<td>305°</td>
<td>−0.49mₙv²a</td>
<td>0.60</td>
<td>0.272mₙv²a</td>
<td>0.252mₙv</td>
</tr>
<tr>
<td>16</td>
<td>22°</td>
<td>239°</td>
<td>239°</td>
<td>327°</td>
<td>−0.38mₙv²a</td>
<td>0.49</td>
<td>0.32mₙv²a</td>
<td>0.29mₙv</td>
</tr>
<tr>
<td>17.5</td>
<td>20°</td>
<td>246°</td>
<td>246°</td>
<td>344°</td>
<td>−0.29mₙv²a</td>
<td>0.41</td>
<td>0.24mₙv²a</td>
<td>0.32mₙv</td>
</tr>
</tbody>
</table>

Conclusions

Nonlinear dynamics of the mechanical system with sectional frictions are discussed in the paper. Based on the theoretical and experimental results, some conclusions are drawn as follows.

The nonlinear dynamic characteristics of the mechanical system with sectional frictions are quite different from those of classical machinery. Based on a typical vibratory cooling machine, the nonlinear force is expressed firstly as considering regional frictions. Then the approximation solutions of the system are deduced by the asymptotic method of nonlinear theory. The combination coefficients and damping coefficients of materials are obtained.

Given the real parameter values of the above cooling machine system, the combination coefficients of materials change from 0.4 to 0.7. The damping coefficients of material change from 0.25 mₙv to 0.33 mₙv, they decrease with increasing amplitudes of the librating body. It is seen that the vibrating system with regional friction can be regarded to have hardness nonlinearity.

The obtained nonlinear dynamic analyses of the system with sectional frictions are very important for designers of these machines.

Acknowledgments

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References