Fractal Features of Surface Morphology of Uneven Eroded Hot-dip Galvanized Steel

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Abstract. The corrosion surface of hot-dip galvanized steel is regarded to be fractal and contains abundant self-similar information. In most cases, the corrosions of surface are not uniform as considered. For such an uneven eroded surface, regional fractals are introduced here in order to improve the analysis accuracy for local corrosions. Basing on the singular value decomposition of surface image, fractal dimensions of the energy norm along with the energy scale are analyzed. Combining energy norm of singular values and wavelet transform, the corrosion surfaces are described in different scales and decomposed levels.

Introduction

Gray images of hot-dip galvanized steel acquired by a metallographic microscope will represent its surface corrosions subtly. In many previous works, statistic parameters are traditionally used to describe metal corrosion surfaces and machining or wearing surfaces. It is important to find some certain parameters that are not relative to any image scales or resolutions [1]. The corrosion morphologies of material surfaces are now known with characteristics of statistic self-similarity or self-mapping [2, 3]. Many investigations with the theory of fractal geometry have been achieved for rough surfaces [4, 5]. For example, the fractal dimension of a film surface is calculated as 1.3 in [6]. In [7], surface gray scale data are used for calculating fractal dimensions that illustrate the corrosion speed and severity quantitatively. The eroded hot-dip galvanized steel surfaces are regarded with fractal characteristics, which can be calculated from its image data, and related directly with erosion speeds, and so on.

However, the fractal dimension obtained according to the entire surface is not always accurate enough if considering non-uniform corrosion cases in practice. In our paper, the surface image is firstly divided into regular regions, and all fractal dimensions of different regions are calculated. Then, fractal dimensions of energy norm along with energy scale are defined for non-uniform corrosion surfaces based on the singular value decomposition (SVD) of image matrix. In addition, according to theory of wavelet transform (WT), an image can be decomposed into approximated and detailed signals, which can illustrate corroded surface morphology in different levels as well.

In this paper, taking the surface images of hot-dip galvanized steel with salt spray corrosions as an example, the regional fractal dimensions are proposed for local corrosions. Fractal dimensions of energy norm via energy scale are calculated based on SVD. Furthermore, WT and energy norm with singular values are also combined to reveal the underlying corrosion theory in a better way.
Elementary Principles

**Regional fractals of corrosion morphology image.** An entire surface image can be divided into some small regions, such as 3X3, based on its size and gray degree. The improved algorithm for box dimensions is adopted for the entire image and all divided regions.

Assume the image size is NXN, where N is the number of pixel points. The gray degrees (256 in total) within it are marked in different size boxes and all the gray degrees of each box are summarized. The fractal box dimension is defined as follows

\[
D = \frac{\ln \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij}}{\ln N}.
\]

where, \( B_{ij} \) is gray level of each box, i and j are the row and column of the image matrix.

**Fractal analysis of corrosion morphology with SVD.** The operation of SVD of a gray matrix \( A \) is unique in mathematics. The singular value vector, \( \alpha_i=(i=1-r) \), of \( A \) are defined as the square roots of \( \lambda_i \) which are the nonnegative eigenvalues of \( AA^T \). Make a direct product of \( U_{nxr} \) and \( V_{nxr} \) corresponding to the \( k \)th singular value and form a subspace \( \psi_k \), and all the \( \psi_k \) compose of a self-contained orthogonal coordinate. The energy density (or energy spectrum radius) of matrix \( A \) projected to \( \psi_k \) is defined as follows

\[
e_k = \alpha_k^2, \quad k = 1 \sim r.
\]

The energy norm (or accumulated energy) in all subspaces corresponding from \( \alpha_i \) to \( \alpha_k \) is

\[
E_k = \sum_{i=1}^{k} \alpha_i^2.
\]

Fractal may exist between the energy norm and energy density above, that is

\[
\ln E_k = D \ln e_k + \ln \beta.
\]

Given \( k = 1 \sim r \), the straight-line slope in double-logarithmic coordinates is the fractal dimension of energy norm via energy density based on the singular vector of gray matrix \( A \).

**Fractal analysis of corrosion morphology based on both SVD and WT.** The expansion formula of \( f(x,y) \in \tilde{V}_0 \) with a two dimensional orthogonal mother wavelet kernel is

\[
f(x,y) = \sum_{j=1}^{l} \sum_{m,n} \left[ \alpha_{m,n,j,m}^{1}(x) \varphi_{j,m}(y) + \beta_{m,n,j,m}^{2}(x) \varphi_{j,m}(y) + \gamma_{m,n,j,m}^{3}(x) \varphi_{j,m}(y) \right] + \sum_{m,n} s_{m,n}^{l}(x) \varphi_{j,m}(y),
\]

where, the superscript \( j \) represents scale, the subscripts represent shifts in two directions. \( \alpha_{m,n,j,m}^{1}, \beta_{m,n,j,m}^{2}, \gamma_{m,n,j,m}^{3} \) are wavelet expansion coefficients in wavelet spaces, \( \tilde{W}_j^1, \tilde{W}_j^2, \tilde{W}_j^3 \), respectively, and \( s_{m,n}^{l} \) is the scale expansion coefficient in scale space \( \tilde{V}_j \).

From the viewpoint of digital filtering, the process of two-dimensional wavelet decomposition is to filter along the row and column directions of a matrix, and four parts of filter output are obtained. The dimensions of four output matrices are all \((N/2)X(N/2)\) if with the input matrix of NXN. The appreciated part can be decomposed into next after once operation of WT and more appreciated and detailed images are obtained for different levels.
Examples and Results

Some samples of hot-dip galvanized steel of $150 \times 100 \times 0.75\text{mm}$ are taken in the DCTC1200P salt spray test box. The corrosive water solution is made from NaCl, $50\pm g/L$, and PH value between 6.5 and 7.2. The surface image sizes by a 53X metallographic microscope are $172 \times 172$ pixels. Figure 1 shows three images with corrosion time of 24 hours, 48 hours and 72 hours.

![Surface images](image)

Figure 1 Three surface images of hot-dip galvanized steel after salt spray corrosion

**Regional fractal dimensions.** According to the double-logarithm curve slopes of above three corrosion images, the calculated box dimensions are 2.402, 2.491 and 2.522, respectively. But all curves of 48h and 72h are so overlapped that the fractals of entire images cannot be used to distinguish from each other.

With divided regions of any images by 3X3, the calculated double-logarithmic curves break away obviously even with different corrosion time. There exist parallel lines in every region for 24 h and 72h, otherwise diverged curves for 48h. That is to say, surface corrosions develop from uniform state into non-uniform state and then into uniform again. The averaged fractal box dimensions are 2.316, 2.453 and 2.498 for the above three cases, respectively.

**Fractals of energy norm based on SVD.** SVD is applied on the above three corrosion morphologies, and their energy densities and energy norms are calculated. In the coordinate system with horizontal axis of energy density and vertical axis of energy norm, their double-logarithmic curves are obtained. All the curves are divided into three segments and fractal dimensions of energy norm for the above three erosion cases are shown in Table 1. The first linearized segments are corresponding to larger singular values and their tangents, i.e. fractal dimension values, are $1.636203$, $1.753067$ and $1.850200$ for the three different time periods, the same meanings with the second and third segments.

Known as smaller singular values are corresponding to the third segments, the fractal dimensions of energy norm of the third segments will indicate the local erosion developing stages.

<table>
<thead>
<tr>
<th>Corrosion time [hour]</th>
<th>1st segment</th>
<th>2nd segment</th>
<th>3rd segment</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1.636203</td>
<td>0.804283</td>
<td>0.730267</td>
</tr>
<tr>
<td>48</td>
<td>1.753067</td>
<td>0.828045</td>
<td>0.674187</td>
</tr>
<tr>
<td>72</td>
<td>1.850200</td>
<td>0.867981</td>
<td>0.637761</td>
</tr>
</tbody>
</table>

**Fractals of energy norm based on SVD at different WT levels.** Here the mother wavelet function is chosen as ‘db5’ given by I. Daubechies and the corrosion surface images are decomposed into 4 levels. For the reconstructed detail images, i.e. high frequency parts, the fractal dimensions of energy norm along energy density based on SVD are calculated according to last section. From the pictures of double-logarithmic curves, those of detailed images of level 2, 3, 4 are all straight lines obviously. The obtained fractal dimensions for different detailed images are listed in Table 2.
Table 2. Fractal dimensions of energy norm based SVD on different wavelet decomposition levels

<table>
<thead>
<tr>
<th>Corrosion time [hour]</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1.002124</td>
<td>1.069274</td>
<td>1.058916</td>
</tr>
<tr>
<td>48</td>
<td>1.004162</td>
<td>1.063036</td>
<td>1.059199</td>
</tr>
<tr>
<td>72</td>
<td>1.002558</td>
<td>1.068072</td>
<td>1.061368</td>
</tr>
</tbody>
</table>

Conclusions

According to the averaged fractal box dimensions of regional corrosion morphologies of hot-dip galvanized steel in neural salt spray, 2.316, 2.453 and 2.498 respectively, the corrosion is much non-uniform when corrosion time is between 24 hours and 48 hours, as examples in the paper. Even the fractal dimensions come from surface images maybe with errors, it could be thought as the easiest way to describe uneven erosion steel surfaces in quantity.

For local corrosions, the fractal dimensions of energy norm corresponding to larger singular values decreases, whereas that of smaller singular values increases. At last, such energy norm fractal dimensions in detailed images from WT are powerful in describing surface eroding process.

In addition, more practical image samples are needed with the above proposal methods to reveal the uneven eroded hot-dip galvanized steel surfaces in a better regularity way.

References