Stability and Bifurcation of Self-Synchronization of a Vibratory Screener Excited by Two Eccentric Motors

Q. K. Han and B. C. Wen

School of Mechanical Engineering and Automation
Northeastern University, Shenyang, China 110004
qhan@mail.neu.edu.cn

Abstract

Varieties of vibratory machines adopting the self-synchronization principle are used in industries, such as vibratory screeners, vibratory conveyors, etc. Normally, two or more exciters with eccentric motors are widely used to drive these machines. The research develops a multi-degree-of-freedom dynamical model for a vibratory screener driven by two exciters rotating in opposite direction considering the mechanical characteristics of eccentric motors. In particular, the differential phase equations of the system are developed for the first time to study the necessary conditions of synchronization. In addition, the stability analysis of the equilibrium points and its bifurcation characteristics of the dynamical system are described via Lyapunov stability theory.

Keywords: Vibratory screener; self-synchronization; differential phase equations; Lyapunov stability; bifurcation

1. Introduction

Synchronization, in its general interpretation, is referred to the correlated or corresponding in-time behaviors of two or more processes, as stated in [1]. From the findings of Huygens, synchronization phenomena have attracted the attentions of many researchers from various research fields [2]. Blekhman [3] firstly studied the mechanism of self-synchronization, i.e. vibration synchronization, with classical nonlinear vibration theories. Nowadays vibratory machines considering synchronization principle have been widely used in industry, such as self-
synchronous vibratory screeners, conveyers and so on [2]. Wen [4] designed various
types of vibratory machines for ore mining industry utilizing the principle of self-
synchronization and his group further investigated classical problems on nonlinear
dynamics modeling of self-synchronizations [5]. Besides, synchronization theories of
chaotic systems have been widely discussed in recent years [6], and the applications
of synchronization of mechanical systems such as different kinds of robots are of
special interests among researchers [7].

It is known that a self-synchronous vibratory machine, usually driven by two or
more eccentric motors with special designed unbalance mass, can achieve a
synchronous vibration state quickly even with initially disturbed rotating speeds and
phases. At the self-synchronous state, the machine body will move only in one
direction, while, the rotating speeds of the two driven exciters are identified and keep
constant phase angle difference. In order to achieve a self-synchronous state, the two
eccentric motors should be chosen and installed properly to couple with the machine
body accordingly. Blekhman firstly presented the self-synchronous analyses of
vibratory machines with dual-excitors [3]. Wen [4, 8] improved the design theories for
self-synchronous machines by investigating the self-synchronization of vibration
machines in two or three dimensions, as well as special phenomenon near resonance.
Recently, some new developments on this research have been reported. Blekhman et
al. developed a general description and expression for self-synchronization and
controlled synchronization [9]. Zhang et al. [10] studied self-synchronous movements
of two rotors driven by two hydraulic motors. The flow of hydraulic motors was
employed to control the rotating phase differences of the two rotors. According to a
dynamical model of electro-mechanical coupling, transition processes of the
synchronous movement of vibratory systems were also simulated in a work [11].

It has to be noticed that although the aforementioned researches on self-
synchronous vibration system have been developed, some issues on nonlinear
dynamical modeling and synchronization stability of the self-synchronous vibration
system have not yet been studied. For example, the effect of exciter’s parameters and
initial conditions on the performance of the vibratory system is still a challenging
topic. In addition, the differential equations governing the phase angle difference of
the two exciters are indispensable for a comprehensive analysis. In this paper, a
vibratory screener is investigated for the analysis of self-synchronous vibration
motions considering mechanical characteristics of two eccentric motors, i.e. their
torques and speeds. Furthermore, equations governing the phase angle differences of
two motors, which are vital to derive the necessary condition of self-synchronous
vibrations of the system, are developed for the first time. Based on the obtained
differential phase equations of the system, the stability of synchronous motion is
studied via Lyapunov stability theory, and the bifurcation characteristics around the
equilibrium points of system are therefore determined.

The organization of the paper is as follow. Section 1 describes the background
and the purpose of this paper. In section 2, the dynamical model of the vibratory
screener with self-synchronization is established. In section 3, the differential phase equations of the system are developed for the synchronization stability and bifurcation analyses. In section 4, analytical results of stability and bifurcations of the system are discussed. Section 5 summaries the results and discussions.

2. Dynamical model of a vibratory screener considering exciters’ motor characteristics

The mechanical model of a vibratory screener with two exciters is shown in Figure 1. The frame of $O_{xy}$ is a global coordinate system. When the machine runs in a self-synchronous vibration state, its mass center $G$ is not the same as origin $O$. The two exciters are driven by two eccentric motors respectively and rotate in opposite directions. The eccentric mass moments of the two exciters are $m_1r_1$ and $m_2r_2$ respectively. Considering the kinetic energy, potential energy and energy dissipation, the dynamical equations of the vibratory screener with two exciters are given as:

$$M \ddot{X} + CX + KX = F$$  \hspace{1cm} (1)

where, $X = [x \ y \ \psi \ \phi_1 \ \phi_2]^T$, $x$, $y$ represent the horizontal and vertical displacements of the machine body; $\psi$ is the swing angle of the body referring to its gravity center; $\phi_1$ and $\phi_2$ are two rotating phase angles of the two eccentric motors, respectively; $M$, $C$, $K$ are the inertial matrix, damping coefficient matrix and stiffness matrix respectively, which are given respectively as follows

$$M = \begin{bmatrix} M & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & J_1 & 0 \\ 0 & 0 & 0 & 0 & J_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_x & 0 & 0 & 0 & 0 \\ 0 & c_y & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & c_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_x & 0 & 0 & 0 & 0 \\ 0 & k_y & 0 & 0 & 0 \\ 0 & 0 & k_y & 0 & 0 \\ 0 & 0 & 0 & k_y & 0 \\ 0 & 0 & 0 & 0 & k_y \end{bmatrix}$$  \hspace{1cm} (2)
In the above expression, \( M = M_0 + m_1 + m_2 \); \( M_0 \) is the mass of the machine body, and \( m_1, m_2 \) are the eccentric masses of the two exciters respectively; \( I = I_0 + m_1 l_1^2 + m_2 l_2^2 \), in which \( I_0 \) is the moment of inertia to the center of \( G \); \( l \) represents the distance between two exciter rotors (\( l = O_1O_2 \)); \( J_i = m_i r_i^2 \), \( i = 1, 2 \), is the moment of inertia of the two exciters; \( r_i \) is the eccentric radius respectively; \( c_x, c_y, c_\psi \) represent damping modulus in the directions of \( x, y, \psi \) of the machine body; \( c = c_\psi + c_1 + c_2 \), and \( c_1, c_2 \) represent the rotary damping modulus of the two eccentric motors; \( k_x, k_y, k_\psi \) are the spring stiffness coefficients of machine body in the directions of \( x, y \) and \( \psi \) respectively.

In Eq. (1), \( F \) is the loading vector, which is described as

\[
F = \{f_1, f_2, f_3, f_4, f_5\}^T
\]

where,

\[
f_1 = m_1 r_1 (\dot{\phi}_1 - \phi_1 \dot{\varphi}_1 + \omega_1 \sin \varphi_1) - m_2 r_2 (\dot{\phi}_2 - \phi_2 \dot{\varphi}_2 + (m_1 - m_2)l(\psi \sin \varphi_1 + \psi \cos \varphi_1)) + (m_1 - m_2)l(\psi \sin \varphi_1 + \psi \cos \varphi_1),
\]

\[
f_2 = m_1 r_1 (\dot{\phi}_1 - \phi_1 \dot{\varphi}_1 + \omega_1 \sin \varphi_1) + m_2 r_2 (\dot{\phi}_2 - \phi_2 \dot{\varphi}_2 + (m_1 - m_2)l(\psi \sin \varphi_1 + \psi \cos \varphi_1) - (m_1 - m_2)l(\psi \sin \varphi_1 + \psi \cos \varphi_1)),
\]

\[
f_3 = -(f_1 \dot{\phi}_1 - f_2 \dot{\phi}_2) + (m_1 - m_2)l(\ddot{x} \sin \varphi_1 - \ddot{y} \cos \varphi_1) - m_1 r_1 l(\dot{\phi}_1 \cos \varphi_1 - \psi \dot{\varphi}_1 - \dot{\psi} \sin \varphi_1 - \psi \cos \varphi_1) + m_2 r_2 l(\dot{\phi}_2 \cos \varphi_2 - \psi \dot{\varphi}_2 - \dot{\psi} \sin \varphi_2 - \psi \cos \varphi_2) - m_1 r_1 g \cos \varphi_1 - m_2 r_2 g \cos \varphi_2,
\]

\[
f_4 = T_1 - f_1 \psi - m_1 r_1 (\ddot{x} \cos \varphi_1 - \ddot{y} \sin \varphi_1) - m_1 r_1 l(\dot{\phi}_1 \cos \varphi_1 - \psi \dot{\varphi}_1 - \dot{\psi} \sin \varphi_1 - \psi \cos \varphi_1) - m_1 r_1 g \cos \varphi_1,
\]

\[
f_5 = T_2 + f_2 \psi - m_2 r_2 (\ddot{x} \cos \varphi_2 + \ddot{y} \sin \varphi_2) + m_2 r_2 l(\dot{\phi}_2 \cos \varphi_2 + \psi \dot{\varphi}_2 + \dot{\psi} \sin \varphi_2 + \psi \cos \varphi_2) - m_2 r_2 g \cos \varphi_2
\]

\( T_1 \) and \( T_2 \) respectively represent the external input torque of two eccentric motors, namely the two driving motors’ output torques.

In Eq. (1), the differential equations of the vibratory screener include inertia moments of two exciters, frictional moments, vibratory moments of the machine body, and gravity moments. According to the fundamental equilibrium principle of torques and moments, the output torques of the two driving motors, \( T_1 \) and \( T_2 \), are always equal to the sum of inertial torques, frictional moments, vibratory moments, and gravity moments and so on. Considering the mechanical characteristics of three-phase electrical motors, the output torques of motors are written as

\[
T_i = 2T_m s_m n_0 n_0 - n_i \frac{n_0 - n_i}{s_m^2 n_0^2 + (n_0 - n_i)^2}, \quad i = 1, 2
\]

where \( n_0 \) is motors’ normal speed; \( T_m \) is the maximum torque of motor, \( T_m = K_T T_N \); \( s_m \) is the critical ratio of motor speed decreasing, \( s_m = s_N (K_T + \sqrt{K_T^2 - 1}) \); \( T_N, s_N \), \( K_T \) are the rated torque, rated ratio of speed decreasing and the rated overload
Stability and bifurcation of self-synchronization

Coefficient respectively. In addition, there exists a relationship between the rotating angular velocity \( \dot{\phi}_i \) and rotating speed \( n_r \) of motor, which is shown as \( \dot{\phi}_i = \frac{n_r}{30} \).

Eqs. (1) - (4) will be employed in the following sections for the analysis of the self-synchronization vibratory screener.

3. Differential equations for phase angles of the vibratory screener

Differential equations for phase angles are developed now for the stability analysis of the self-synchronization vibratory screener. According to Eq. (1), we obtain the dynamical equations of the vibratory screener as follows by ignoring efforts of small damping and higher order small terms

\[
M\ddot{x} + c_1 \dot{x} + k_1 x = m_1 \phi_1^2 \gamma_1 \cos \phi_1 - m_2 \phi_2^2 \gamma_2 \cos \phi_2,
\]

\[
M\ddot{y} + c_1 \dot{y} + k_1 y = m_1 \phi_1^2 \gamma_1 \sin \phi_1 + m_2 \phi_2^2 \gamma_2 \sin \phi_2.
\]

\[
I\ddot{\psi} + c_\psi \dot{\psi} + k_\psi \psi = m_1 \phi_1^2 \gamma_1 l \sin \phi_1 + m_2 \phi_2^2 \gamma_2 l \sin \phi_2.
\]

The stable displacement solutions for the vibrating body in \( x, y \) and angular \( \psi \) directions are supposed to be \( x = A_1 \cos \phi_1 - A_2 \cos \phi_2, \ y = B_1 \sin \phi_1 + B_2 \sin \phi_2, \ \psi = C_1 \sin \phi_1 - C_2 \sin \phi_2 \). The aforementioned stable solution expressions are substituted into Eq. (6) to obtain the amplitudes of approximate solutions of the system as follows

\[
A_1 = \frac{-m_1 \phi_1^2}{m_2 \phi_2^2 - k_2}, \quad B_1 = \frac{-m_1 \phi_1^2}{m_2 \phi_2^2 - k_2}, \quad C_1 = \frac{-m_2 \phi_2^2 l}{I \phi_1^2 - k_\psi}.
\]

Only considering the two eccentric rotors’ dynamical equations, and ignoring the higher order terms of \( m_1 r_1 l \ddot{\psi} \cos(\phi_1 - \psi) \) and \( -m_2 r_2 l \ddot{\psi} \cos(\phi_2 + \psi) \), the simplified equations of the two rotors are obtained as

\[
J \ddot{\phi}_i + c_i \dot{\phi}_i + m_i r_i \gamma_i \cos \phi_i + m_i r_i (\ddot{y} \cos \phi_i - \dot{x} \sin \phi_i)
\]

\[
- m_1 r_1 l \ddot{\psi} \sin(\phi_1 - \psi) = T_1
\]

\[
J \ddot{\phi}_2 + c_2 \dot{\phi}_2 - c_1 \phi_1 - m_2 r_2 \gamma_2 \cos \phi_2 + m_2 r_2 (\ddot{y} \cos \phi_2 + \dot{x} \sin \phi_2)
\]

\[
+ m_2 r_2 l \ddot{\psi} \sin(\phi_2 + \psi) = T_2.
\]

Herein, the expressions for the rotors’ phase angles and their derivatives are introduced by

\[
\phi_i = \Theta \tau + \alpha_i = \tau + \alpha_i,
\]

\[
\dot{\phi}_i = \frac{d \phi_i}{dt} = \frac{d(\tau + \alpha_i)}{d\tau} \frac{d\tau}{dt} = \Theta (\tau + \alpha'_{\tau}) , \ i = 1, 2
\]

\[
\ddot{\phi}_i = \frac{d^2 \phi_i}{dt^2} = \frac{d[\Theta (\tau + \alpha'_{\tau})]}{d\tau} \frac{d\tau}{dt} = \Theta^2 \alpha''_{\tau}.
\]
where, \( \tau = \omega_t \), \( \omega_t \) is two eccentric motor's equivalent average rotating speeds; \( \alpha_i \) is phase angle of rotors, and \( \tau \gg \alpha_i \) because \( \tau \) is the multiple of running time. The symbol ( )' represents \( \frac{d}{d\tau} \) and ( )'' represents \( \frac{d^2}{d\tau^2} \). Substituting \( A_i \), \( B_i \), \( C_i \) in Eq. (7) and \( \phi_i \), \( \phi_i \) in Eq. (9) into Eq. (8), the equations of \( \alpha_i \) can be obtained after conducting Fourier transform and integrating \( \tau \) in its region of \([0,2\pi]\), and the first equation of Eq. (8) becomes

\[
J_2\ddot{\alpha}_2^* + c_i\ddot{\alpha}_i(1+\alpha'_i) + \delta - T_i = 0
\]

where

\[
\delta = \frac{m_i r_i^2}{2} \left[ -\frac{m_i r_i [\sigma(1+\alpha'_i)]^2}{m_i [\sigma(1+\alpha'_i)]^2 - k_x} + \frac{m_i [\sigma(1+\alpha'_i)]^2}{m_i [\sigma(1+\alpha'_i)]^2 - k_y} \right] \\
- \frac{\cos(\alpha_i - \alpha_z) m_i r_i r_i [\sigma(1+\alpha'_i)]^2 \dddot{\alpha}_2^*}{2m_i [\sigma(1+\alpha'_i)]^2 - 2k_y} - \frac{\sin(\alpha_i - \alpha_z) m_i r_i r_i [\sigma(1+\alpha'_i)]^4}{2m_i [\sigma(1+\alpha'_i)]^2 - 2k_y} \\
+ \frac{\cos(\alpha_i - \alpha_z) m_i r_i r_i [\sigma(1+\alpha'_i)]^2 \dddot{\alpha}_2^*}{2m_i [\sigma(1+\alpha'_i)]^2 - 2k_x} + \frac{\sin(\alpha_i - \alpha_z) m_i r_i r_i [\sigma(1+\alpha'_i)]^4}{2m_i [\sigma(1+\alpha'_i)]^2 - 2k_x}
\]

(11)

In the self-synchronization process, the two eccentric rotors rotate periodically and the integration of \( \dddot{\alpha}_2^* \) in \([0,2\pi]\) is zero. Therefore, \( \delta \) becomes

\[
\delta = \frac{\sin(\alpha_i - \alpha_z) m_i r_i r_i [\sigma(1+\alpha'_i)]^4}{2m_i [\sigma(1+\alpha'_i)]^2 - 2k_y} - \frac{\sin(\alpha_i - \alpha_z) m_i r_i r_i [\sigma(1+\alpha'_i)]^4}{2m_i [\sigma(1+\alpha'_i)]^2 - 2k_x}
\]

(12)

Similarly, the differential equation of phase angle \( \alpha_2 \) can be derived from Eq. (8) as

\[
J_2\ddot{\alpha}_2^* + c_i\ddot{\alpha}_i(1+\alpha'_i) - \delta - T_z = 0
\]

(14)

The parameters of the two eccentric rotors are assumed to be identical, namely,
\[ m_1 = m_2 = m, r_1 = r_2 = r, c_1 = c_2 = c. \] By setting \( \Delta T = T_i - T_z \), \( \Delta k = k_x - k_y \), and

\[
\delta = \frac{1}{2} \frac{m_i r_i r_i^2}{M^2} (k_x - k_y) \sin(\alpha_i - \alpha_z)
\]

(13)
\[ \alpha = \alpha_1 - \alpha_2, \] the differential equation of the phase difference \( \alpha \) for the state of self-synchronous vibration is finally obtained as
\[ J \dddot{\alpha} + c \dot{\alpha} + H_0 \sin \alpha = \Delta T \] (15)
where, \( J = mr^2 \); \( \alpha_1, \alpha_2 \) are the rotating angles; \( c = c_1 = c_2 \) is the rotating damping of the two eccentric motors; \( T_1 \) and \( T_2 \) are the torques of the two motors. \( H_0 \) in Eq. (15) can be expressed as
\[ H_0 = \left( \frac{mr^2}{M} \right) \Delta k \] (16)
where, \( \Delta k = k_x - k_y \), \( k_x \) and \( k_y \) are the stiffness of machine body in \( x \) and \( y \) directions respectively.

4. Stability analysis and bifurcations of the vibratory screener

Based on the developed dynamical equations of the vibratory screener and the corresponding differential equation of the phase difference \( \alpha \) of eccentric motors, the stability analysis and bifurcations of the vibratory screener for self-synchronization are conducted below.

4.1 Necessary condition for self-synchronization of the machine body

Let \( x_1 = \alpha \) and \( x_2 = \alpha' \) be set, Eq. (15) can be re-written as
\[
\begin{cases}
 x_1' = x_2 \\
 x_2' = -\frac{c}{J} x_2 - \frac{H_0}{J} \sin x_1 + \frac{\Delta T}{J} \omega^2
\end{cases}
\] (17)

The corresponding equilibrium point equations are thus obtained to be
\[
\begin{cases}
 x_1 = 0 \\
 -\frac{c}{J} x_2 + \frac{1}{J} \omega^2 (\Delta T - H_0 \sin x_1) = 0
\end{cases}
\] (18)

It is obvious from Eq. (18) that the system of the vibratory screener has no equilibrium point if \( -H_0 \sin x_1 + \Delta T \neq 0 \), and hence the system cannot achieve self-synchronous vibration at all. Meanwhile, although the condition \( -H_0 \sin x_1 + \Delta T = 0 \) may be satisfied, the phase angle difference \( \alpha \) does not exist, and the system has no equilibrium point either if \( \frac{\Delta T}{H_0} > 1 \).

Hence, \( \sin x_1 = \frac{\Delta T}{H_0} \) and \( \frac{\Delta T}{H_0} \leq 1 \), should be satisfied in order to gain steady synchronization states. From this point of view, the necessary condition of achieving self-synchronous vibrations is provided as
\[
\left(\frac{mr}{M}\right) \Delta k \geq \Delta T \tag{19}
\]

Figure 2 shows the relationship between \(\Delta T\) and \(\frac{mr}{M}\) according to the necessary condition of Eq. (19) for self-synchronous vibrations of the screener, where \(\Delta k\) is a constant. From the figure, it concludes that if the parameter values are within the shadow regions in Fig. 2, the machine body is eligible to induce a self-synchronous vibration for a vibratory screener.

**4.2 Stability and bifurcations**

According to the differential equation for the phase difference \(\alpha\) of the two eccentric motors, i.e. Eq. (15) or Eq. (17), the stability and bifurcations of equilibrium points of the self-synchronous system for the vibratory screener are discussed in this section. Here the Lyapunov stable theory based on the first order approximate system of Eq. (17) is employed. Following the Lyapunov stable theory, if the real part of an eigenvalue of the first order approximate system is negative, the zero order solution of the system is asymptotically stable. If there is one positive real part at least, the zero order solution is unstable. Otherwise, if a real part of an eigenvalue is zero and the others are negative, the stability of the zero order solutions are not confirmed and the stability of system needs to be verified by the nonlinear items of the system.

Hence, for the self-synchronous system of the vibratory screener discussed here, the stability of synchronous vibrations at the equilibrium point \((\text{arcsin} \frac{\Delta T}{H_0}, 0)\) of Eq. (17) is obtained as follows. The first order approximate equation from Eq. (17) is given below

\[
\begin{bmatrix}
    x_1' \\
    x_2'
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 \\
    -\frac{H_0 \cos x_1}{J\bar{\omega}^2} & -\frac{c}{J\bar{\omega}}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
\tag{20}
\]

The corresponding eigenvalue equation of Eq. (20) is
\[ \begin{vmatrix} -\lambda & 1 \\ \frac{H_0 \cos x_i}{J \omega^2} & -c/J \omega - \lambda \end{vmatrix} = 0 \] (21)

Namely,

\[ \lambda^2 + \frac{c}{J \omega} \lambda + \frac{H_0 \cos x_i}{J \omega^3} = 0 \] (22)

From Eq. (22) we know that, if \( x_i = \alpha \) and \( \cos \alpha > 0 \), i.e. \( 0 \leq \alpha < \pi/2 \), the eigenvalue determined by Eq. (22) has a negative real part, and the equilibrium point of Eq (17), i.e. \( \arcsin \left( \frac{\Delta T}{H_0} \right), 0 \), is stable according to the Lyapunov theory. On the other hand, if \( \cos \alpha < 0 \), i.e., \( \pi/2 < \alpha < \pi \), the eigenvalue has a positive real part which leads the equilibrium point to be unstable. Finally, if \( \cos \alpha = 0 \), i.e. \( \alpha = \pi/2 \) and \( \frac{\Delta T}{H_0} = 1 \), the system stability cannot be determined. For the bifurcations of the self-synchronous system, it is obviously that the number of solutions of Eq. (22) will change from two to one if \( \frac{\Delta T}{H_0} \) varies from 0 to 1. A saddle-node bifurcation takes place at \( \frac{\Delta T}{H_0} = 1 \). For the case of having two solutions of Eq. (22), the upper solution branch is unstable, as shown in Figure 3. Finally, the system will have no solutions if \( \frac{\Delta T}{H_0} > 1 \).

![Figure 3. Stable and unstable solution branches of Eq. (22).](image)

In addition, for the vibratory screener discussed in the paper, the output torques of eccentric motors are dependent on their rotating speeds. Considering the fact that the mechanical characteristics of the two motors are the same and the rotating speeds are close to their rated speeds, the difference between the two motors’ output torques is proportional to that of the phase angles. In this case, the corresponding equilibrium point of the system is \((0, 0)\). The equilibrium point of \((0, 0)\) is stable because the real parts of two eigenvalues of Eq. (22) are negative.
5. Simulations of stable self-synchronizations and nonsynchronizations

In order to validate the analysis of the above self-synchronous system, numerical simulations are carried out hereinafter, based on the dynamical model of Eq. (1), not the simplified model in Eq. (15). The following parameters of a practical vibratory screener are employed in the simulations, \( M \approx 130 \text{kg} \), \( m_1 = m_2 = 2.5 \text{kg} \), \( r_1 = r_2 = 0.08 \text{m} \), \( l = 0.4 \text{m} \), \( I = 33 \text{kgm}^2 \), \( k_x = 3000 \text{N/m} \), \( k_y = 77600 \text{N/m} \), \( k_\psi = 3000 \text{Nm/rad} \). The parameters of two eccentric motors are \( T_{N1} = T_{N2} = 2 \text{Nm} \), \( f_N \approx 16 \text{Hz} \), \( s_{N1} = s_{N2} = 5\% \) and \( K_{T1} = K_{T2} = 2.5 \). In addition, the approximate values of damping coefficients are \( c_x = c_y = 100 \text{Ns/m} \), \( c_\psi = 100 \text{Nms/rad} \), \( c_1 = c_2 = 0.01 \text{Nms/rad} \).

As discussed in section 4, since the eccentric motors of the vibratory screener can be adjusted, the simulations are conducted at given mass, stiffness, and damping parameters but with variation of the difference of torques \( \Delta T \). Two typical simulation results are illustrated in Figure 4 for different values of \( \Delta T \) selected from different parameter regions shown in Figure 2 and Figure 3 for stable and unstable state of the system.

In Figure 4(a), the machine body of the vibratory screener is in a steady self-synchronous vibration state when \( 0.001 \text{Nm} \). In this case, the body vibration in horizontal direction is very small, and the phase angle difference of the two eccentric motors retains at an unchangeable value. Figure 4(b) shows the simulation results of a non-synchronous vibration of the machine body, where \( 0.08 \text{Nm} \) is selected in the unstable region. At the non-synchronization state, the magnitudes of vibrations are very big in both \( x \) and \( y \) directions. The phase angle difference between the two eccentric motors increases monotonically.

![Figure 4(a)](image)

(a) Displacements in \( x, y \) directions of machine body and motors’ phase angle difference when \( \Delta T = 0.001 \text{Nm} \)
The simulations on the effect of $\Delta T$ on the stability of the self-synchronous system verify the theoretical analysis in section 4. Similarly, the effect of the exciter’s eccentric mass moments to the machine body mass ratio, $\frac{mr}{M}$, has great effect on the synchronous vibrations of the vibratory screener. If the ratio value $\frac{mr}{M}$ is too large, the inequality of Eq. (19) is not satisfied and hence the stable self-synchronization cannot be achieved.

Figure 5(a) shows the vibration of the vibratory screener at $m_1r_1 = m_2r_2 = 0.2$kgm. The magnitude in $y$ direction is 2.67mm and 3.98mm respectively at the stable self-synchronization state.

When the eccentric mass moments become bigger, the magnitude in $y$ direction is bigger and the transient time for achieving a stable state is shorter. However, a self-synchronous motion cannot be achieved when the momentum is beyond a critical value, for example, $m_1r_1 = m_2r_2 = 0.4$kgm, in the simulation shown in Figure 5(b). The phase angle difference between the two eccentric motors is again found to increase monotonically showing an unstable synchronization motion, i.e. non-synchronization.
Q. K. Han and B. C. Wen

6. Conclusions

The dynamical model of a vibratory screener with two mass-eccentric exciters is developed in this paper considering the two exciters’ eccentric motor characteristics. Based on the established dynamical model, a differential equation about the phase angle difference of the two eccentric motors driving the machine is developed for the first time for investigating the stable analysis for its self-synchronous vibrations. Numerical simulations according to Eq. (1) confirm the theoretical analyses.

From the differential equation of phase angle difference of the screener, the necessary condition for self-synchronization and the stability analysis of synchronous vibration are obtained. According to the stability analysis, the system is under a critical state if \( \frac{\Delta T}{H_0} = 1 \) where a saddle-node bifurcation occurs. If \( \frac{\Delta T}{H_0} > 1 \), there are two solutions, in which the upper solution branch is unstable and the lower solution branch is stable self-synchronization. Numerical simulations have been carried out to reveal and verify the stable self-synchronizations with typical system parameters in the stable region.

Acknowledgements

This work is supported by the Key Project of Science and Technology of Chinese Ministry of Education (No: 108037), and the National Nature Science Funds of China (Granted No.10402008 and No.50535010). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.
References


Received: April 2, 2008