Periodic motions of a dual-disc rotor system with rub-impact at fixed limiter

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Abstract: Periodic motions of a rotor system with two discs are investigated where rub-impacts occur at fixed limiter for a test rig with dual discs. First, a finite element (FE) model of the rotor system is developed. Then numerical simulations based on the FE model are conducted to study the rotor transverse vibrations of the rotor system under three typical cases with different rotating speeds, rub-impact clearances, rub-impact rod stiffness, and rub frictions. The results are further compared with typical multiperiodic characteristics by experimentally measured vibrations. The simulations demonstrate different rotor motions, including periodic, quasi-periodic or complex characteristics, which coincide with experimental measurements. Finally, the non-stationary time-frequency domain characteristics of rub-impact motions are investigated via the Hilbert–Huang transform, and intrinsic mode functions (IMFs) and instantaneous frequencies of the three typical cases are obtained. The research has revealed some of the inherent vibration features of the dual-disc rotor system with rub-impact only occurring at fixed limiters.

Keywords: rotor system, rub-impact, fixed limiter, multiperiodic motions, finite element model

1 INTRODUCTION

Due to the increasing demand for high speed and high efficiency, the clearance between rotors and stators in modern rotating machinery is becoming smaller and smaller. As a result, the rub-impact, which refers to the contact between the rotating and non-rotating structures in a machine, has become one of the most common damaging malfunctions of rotating machinery \cite{1}. Further investigation on the mechanism of the rub-impact phenomenon and its dynamic characteristics is indispensable for the improvement of the current diagnosis of rotor systems.

Intensive research has been conducted on rub-impact related dynamical phenomena of rotor–stator systems. Many earlier important results were summarized by Muszynska \cite{2}. Transient analysis, thermal coupling, and eliminated activities were investigated in some recent work. In reference \cite{3}, the transient response of an overcritical high-speed rotor was studied, where rapid increases of unbalance initiate from separate collisions to full annular rub. Popprath and Ecker \cite{4} presented a Jeffcott rotor model with intermittent contact with a stator interacting with the rotor model via non-linear contact forces. The sub-model of the stator was modelled as an additional vibratory system. Bachschmid \textit{et al.} \cite{5} studied the thermally-induced spiral vibrations using a fully assembled machine model (rotor, bearings, and foundation) and implemented a sophisticated thermal and contact model with one-dimensional finite beams and a three-dimensional model for temperature distribution analysis. Banakh and Nikiforov \cite{6} studied vibro-impact interaction between rotor and floating sealing ring, where hydrodynamic forces in the clearance between the rotor and the ring, as well as dry friction between the ring and the casing, were taken into account. In the work of Jiang \textit{et al.} \cite{7}, a new method was presented to increase the rotating machinery's adaptive capability to the rubbing conditions through active auxiliary bearings.
It is well known that rotor systems with rub-impacts behave non-linearly due to discontinuous contacts between rotors and stators, and they often show very complicated vibrating phenomena, such as periodic motion, quasi-periodic motion, and even chaotic motions. Ehrich [8] observed the higher order sub-harmonic vibrations as well as chaotic vibration in a high-speed turbomachine. Goldman and Muszynska [9] investigated the supercritical subharmonic phenomenon and the chaotic behavior of rotor–stator systems with rubs via an analytical approach associated with the numerical calculation. Piccoli and Weber [10] reported chaotic motions of a rubbing rotor system in experiments. Chu and Zhang [11] discussed some typical periodic, quasi-periodic, and chaotic motions with the chaos-bifurcation theory for a Jeffcott rotor system with rub-impact. In another paper by Chu [with Lu, 12], some experiments were conducted in a full rubbing rotor system with various forms of periodic and chaotic vibrations, containing multiple harmonic components and 1/2 fractional harmonic components, even with 1/3 fractional harmonic components. The rich mixtures of motions for rotor systems with rub-impact manifest themselves in the occurrence of multiple solutions in the viewpoint of mathematics.

As it is known, there are different kinds of rubbing effects in rotor systems. The most frequently occurring case is the rub-impact between rotor shafts and seals and/or bearings. A rub-impact may also take place around the journal surface with either partial or full circular contacts. From experiments on aero-engine test rigs [13], it has been observed that often rubbings only occur at fixed points along the axial direction if they are caused by the deformation of compressor cases. In the present work, the multiperiodic motions of a dual-disc rotor system with rub-impact on fixed limiters are simulated via a developed finite element (FE) model. In addition, a dual-disc rotor system based on a real test-rig is also developed. This system is not a traditional Jeffcott rotor with only one disc and it takes into account specific effects such as the coupling influence from distributed mass along the rotor shaft.

In rotor dynamics, the classical lumped mass rotor models are useful for qualitative analyses. The transfer matrix technique, however, will not be powerful for non-linear dynamics and fault dynamics of a rotor system. Nelson and McVaugh [14] introduced the rotor dynamical analysis with FE method, first. In Lalanne and Ferraris’ book [15], the rotor dynamics were fully discussed with FE models. Then, in a book by Kicinski [16], non-linear modelling and responses were explained in detail based on FE method for rotor-bearing-support systems of turbine generator units. It has been well acknowledged that the validation of any developed FE model is indispensable, and the model parameters should first be identified with experimental measurements [17].

In the simulations of multiperiodic motions of a dual-disc rotor system with rub-impact at given limiters, a well calibrated FE model is further developed in this paper with Euler-Bernuli beam elements and the consideration of gyroscopic effects of the discs for the rotor system. The bearings are modelled by linear stiffness and dampings with identified values. The rub-impacts are simulated using an elastic rod to produce contact deformation in a horizontal or vertical direction at a given location along the rotor shaft. Based on the model, vibrations are analysed for the rotor system with various rotational speeds, impact stiffness and clearances. Characteristics of different periodic motions under different operating cases are also studied. The vibrations via the FE model are compared with those measured in experiments.

Traditional analysis methods, such as rotor trajectory and power spectrum, were commonly used to describe the multiperiodic vibrations of a rotor system. Since the rub-impact rotor system has a non-stationary and non-linear nature, it seems that fast Fourier transform (FFT)-based methods are no longer sufficient to describe the inherent information. Wavelet transform (WT), being regarded as a multiscale analysis tool for signals through dilation and translation, has been widely used recently. However WT has its own inevitable deficiencies, including interference terms, border distortion, and energy leakage, all of which will generate small undesired spikes throughout the frequency scales, making the results confusing and difficult to interpret. A novel time-frequency analysis named the Hilbert–Huang transform (HHT) method has become acceptable because of its uniform resolution at the low- and high-frequency parts [18, 19]. In the last part of this paper, the simulated responses of the rub-impact rotor system are analysed using HHT to show their complex periodical characteristics. This paper consists of five sections. After this introduction, an FE dynamical model is established for a dual-disc experimental rotor system in section 2. In section 3, simulations are conducted for different periodic motions patterns and the results are compared among themselves and with measured vibrations from experiments. In section 4, in order to identify the non-stationary and non-linear characteristics motions of the rotor with rub-impact, typical vibration responses from simulations are analysed with HHT. Finally, conclusions are drawn in section 5.

2 MODEL DEVELOPMENTS AND PERIODIC MOTIONS ANALYSIS

In order to investigate the dynamical behaviour of the rotor system with rub-impact at fixed limiters, a simple test rig is used here, as illustrated in Fig. 1.
This rotor system consists of a flexible shaft supported by two journal bearings. Two discs are mounted along the shaft. The rotor is driven by a small motor with frequency control. Three eddy current probes are used to measure the vibration of the rotor in both x and y directions and the sample frequency is 5 kHz. The location of rub-impact with the fixed limiter can be on the shaft or disc. The model to be established in this paper can be applied to both of these cases.

Some simplifications are introduced as follows:

(a) The rotor is flexible and it can be modelled by an Euler–Bernulli beam with distributed mass and elasticity, and two discs with larger diameters;
(b) The bearings are linearized ideally with stiffness and damping;
(c) The supports and/or foundations are rigid;
(d) The torsional movements are negligible.

2.1 FE model of the rotor system with rub-impact at fixed limiters

The shaft is divided into 14 beam elements and the number of total nodes is 15, as shown in Fig. 2. Segments 5–6 and 11–12 are the two discs, nodes 1 and 15 are the two bearing locations, and some unbalance masses can be fixed at nodes 5 and/or 11.

Each node of the beam element has two translational degrees of freedom and two rotating degrees of freedom for bending analysis of the rotor without axial deformations. A beam element is shown in Fig. 3, where the local coordinate system is regarded as the same as the global system in a simple way.

The general displacement vector of a beam element for the shaft, \( \mathbf{u}^e \), is given as

\[
\mathbf{u}^e = \begin{bmatrix} u_A & v_A & \theta_{x_A} & \theta_{y_A} \\
 u_B & v_B & \theta_{x_B} & \theta_{y_B} \end{bmatrix}^T
\]  

The two sliding bearings at the two ends of the shaft are modelled as rigid boundaries. According to the previous identification studies of the rotor system by the Han and Wen et al.\[20]\, the stiffness of the bearings is approximately \( 9 \times 10^6 \) N/m in both horizontal and vertical directions. Because of the large stiffness of the bearings, it is reasonably postulated that the rotor shaft is constrained for all transverse degrees of freedom (DOFs) but not for rotating DOFs. Usually the axial DOFs can also be ignored due to their weak influence on rotor transverse vibration analyses. The identified damping coefficients are found to be \( 1.5 \times 10^3 \) N/(m/s) for both the horizontal and vertical directions of the bearing. The supporting bases are all certainly constrained with six DOFs.

Taking into account the influence of the transverse and rotating moments of inertia and the gyroscopic effects on the shaft element based on the above-mentioned FE model, including shaft with discs, unbalances, rub-impacts, and boundary conditions, the general non-linear motion equations of the entire
system are written as
\[ M\ddot{u} + D\dot{u} + Ku = F_g + F_u + F_{\text{rub}} \]  
where \( u \), the nodal displacement matrix, i.e., general coordinate of the FE system, is directly shown as
\[ u = [u_1, v_1, \theta_{x_1}, \theta_{y_1}, u_2, v_2, \theta_{x_2}, \theta_{y_2}, \ldots, u_n, v_n, \theta_{x_n}, \theta_{y_n}]^T \]  
The gravity vector of the beam element, \( F_g \), is provided to be
\[ F_g = [0, -g, 0]^T \]  
The unbalances are assumed to exist only at a few nodes, such as at disc 1 and/or disc 2. The unbalance force vector of element due to one eccentric mass, \( F_u \), is thus shown as
\[ F_u = [m_c e \Omega^2 \cos \Omega t, m_c e \Omega^2 \sin \Omega t, 0, 0]^T \]  
The total mass and stiffness matrices, \( M \) and \( K \), are assembled with every element mass matrices, \( M_e \), and element stiffness matrices, \( K_e \). \( M_e \) and \( K_e \) come from the beam element patterns introduced by Nelson \[14, 21\]. \( M_e \) includes the translating inertial matrix, \( M_{\text{Tr}} \), and rotating inertial matrix, \( M_{\text{Rot}} \), of an element. \( M_e \) and \( K_e \) are given as follows
\[ M_e = \frac{\rho A l}{420} \begin{bmatrix} 156 & 0 & 0 & 22l & 54 & 0 & 0 & -13l \\ 0 & 156 & -22l & 0 & 0 & 54 & 13l & 0 \\ 0 & -22l & 4l^2 & 0 & 0 & -13l & -3l^2 & 0 \\ 22l & 0 & 0 & 4l^2 & 13l & 0 & 0 & -3l^2 \\ 54 & 0 & 0 & 13l & 156 & 0 & 0 & -22l \\ 0 & 54 & -13l & 0 & 0 & 156 & 22l & 0 \\ 0 & 13l & -3l^2 & 0 & 0 & 22l & 4l^2 & 0 \\ -13l & 0 & 0 & -3l^2 & -22l & 0 & 0 & 4l^2 \end{bmatrix} \]  \[ (6) \]
\[ K_e = \frac{EI}{l} \begin{bmatrix} 12 & 0 & 0 & 6l & -12 & 0 & 0 & 6l \\ 0 & 12 & -6l & 0 & 0 & -12 & -6l & 0 \\ 0 & -6l & 4l^2 & 0 & 0 & 6l & 2l^2 & 0 \\ 6l & 0 & 0 & 4l^2 & -6l & 0 & 0 & 2l^2 \\ -12 & 0 & 0 & -6l & 12 & 0 & 0 & -6l \\ 0 & -12 & 6l & 0 & 0 & 12 & 6l & 0 \\ 0 & -6l & 2l^2 & 0 & 0 & 6l & 4l^2 & 0 \\ 6l & 0 & 0 & 2l^2 & -6l & 0 & 0 & 4l^2 \end{bmatrix} \]  \[ (8) \]
In equation (2), \( D \) is the total damping matrix and assembled by every element of the rotor system. Each beam element includes viscous damping, bearing damping, and gyroscopic moments. The gyroscopic moment matrix of a beam element, \( G_e \), is given as
\[ G_e = \frac{2\rho Ar^2}{120l} \begin{bmatrix} 0 & -36 & 3l & 0 & 0 & 36 & 3l & 0 \\ 36 & 0 & 0 & 3l & -36 & 0 & 0 & 3l \\ -3l & 0 & 0 & -4l^2 & 3l & 0 & 0 & 4l^2 \\ 0 & -3l & 4l^2 & 0 & 0 & 3l & -4l^2 & 0 \\ 0 & 36 & -3l & 0 & 0 & -36 & -3l & 0 \\ -36 & 0 & 0 & -3l & 36 & 0 & 0 & -3l \\ -3l & 0 & 0 & 4l^2 & 3l & 0 & 0 & 4l^2 \end{bmatrix} \]  \[ (9) \]
\( C \), the viscous part of the total damping matrix, \( D \), is the weighted summation of \( M \) and \( K \), as
\[ C = \alpha M + \beta K \]  \[ (10) \]
where
\[ \alpha = \frac{2(\xi_2/\omega_{n2} - \xi_1/\omega_{n1})}{1/\omega_{n2}^2 - 1/\omega_{n1}^2} \]
\[ \beta = \frac{2(\xi_2/\omega_{n2} - \xi_1/\omega_{n1})}{\omega_{n2}^2 - \omega_{n1}^2} \]  \[ (11) \]
where \( \xi_1 \) and \( \xi_2 \) are corresponding coefficients that can be taken as 0.05 and 0.08, respectively.
In equation (2), the rub-impact force vector at a node \( i \) is
\[ F_{\text{rub},i} = [F_x, F_y, 0, 0]^T \]  \[ (12) \]
The functions \( F_x \) and \( F_y \) will be introduced in the following section.

### 2.2 Rub-impact modelling
It is known that the rub-impact only occurs at the fixed limiter due to large vibrations of the rotating shaft or
The Heaviside function is shown as

\[ \text{Heaviside}(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \]

The Young's modulus is \( E = 2.06 \times 10^5 \text{MPa} \). The first natural frequency is \( n_1 = 1026.78 \text{rad/s} \) (i.e. 163.50 Hz (9810 r/min)) and second natural frequency of the rotor system \( n_2 = 277.11 \text{rad/s} \) (i.e. 44.126 Hz (2647 r/min)) and \( n_3 = 163.50 \text{Hz} \) (9810 r/min), respectively. The rotating speeds of the test rig are all within the range from the first critical speed to the second. The rotor system can be considered a flexible system with two rigid supports. The damping ratios, rub-impact stiffness, and friction coefficient are given empirically. In order to verify the FE model simulation results, some experimental measurements are also shown in corresponding running cases. In experiments, at

2.4 Stable analysis of the multi-periodic motions of the rotor system

Multiperiodic motions of a rotor system with rub-impact can be analysed by the harmonic balance method (HBM) [23]. Before analytically processing, the number of DOFs of the dynamic FE system (shown in equation (2)) must be reduced. A simple way is to use modal decomposition under the assumption of local non-linearity of rub-impact. For the rotor system with only a few DOFs, HBM can be used for analytical responses. In this way, equation (2) can be reduced and written into Fourier expansions. The non-linear rub-impact force terms are also expanded into Fourier series. After comparing the coefficients of \( \cos j\tau \) and \( \sin j\tau \) on both of its sides and obtaining the zeroth and \( j \)th orders' algebra equations with integration terms, the inverse Broyden rank-one iteration method can be used to solve the non-linear equation group with good global convergence.

After Fourier expansion, the Floquet theory can be used to provide qualitative analysis of stability and bifurcations of the obtained periodic motions. The stability analysis for the dual-disc rotor with rub-impact at limiters indicates that there exists a stable periodic solution for this system. The available research findings in reference [11] demonstrate that there also exists period-doubling bifurcation when the rotating speeds are at certain values.

3  SIMULATIONS OF MULTI-PERIODICAL MOTIONS OF THE ROTOR SYSTEM

Transverse vibrations of each rotor system node are simulated with different rub-impacts. The direct integration method, Newton–Raphson technique, is employed in the FE model. In simulations, the rotor system parameters are the same as the practical test rig in section 2. The lengths of each shaft segment are 40, 40, 10, 20, 10, 40, 40, 40, 10, 20, 10, 40, 40 mm, respectively. The diameters of every shaft segment are 10 mm. The diameter and width of the two discs are 80 mm and 20 mm. The Young's modulus is \( E = 2.06 \times 10^5 \text{MPa} \). The first and second natural frequencies of the rotor system are calculated and found to be \( \omega_{n1} = 277.11 \text{rad/s} \) and \( \omega_{n2} = 1026.78 \text{rad/s} \) (i.e. 44.126 Hz (2647 r/min) and 163.50 Hz (9810 r/min), respectively). The operating speeds of the test rig are all within the range from the first critical speed to the second. The rotor system can be considered a flexible system with two rigid supports. The damping ratios, rub-impact stiffness, and friction coefficient are given empirically.
different rotating speeds and different local rubbing cases, horizontal and vertical shaft vibrations of the test rig are measured with eddy proximity probes, including nodes near two bearings, two discs, and other interesting nodes.

The following three vibration motions of the rotor system will be discussed. The first is the vibration of the system with a lower rotating speed of 57.5 Hz and weak rub-impact, in which partly distorted ellipses of shaft centre orbits are shown. The second is the vibration of the system with a rotating speed of 70 Hz and a bigger friction coefficient of rubbing, in which double periodic ellipses of shaft centre orbits are shown. The third is the vibration of the system with a higher rotating speed of 82 Hz. Three typical multi-periodic vibrations of shaft including periodic, double periodic, and quasi-periodic vibrations are shown. The simulation results are in good agreement with the experimental measurements.

3.1 Case 1

The rotating speed is 57.5 Hz. The eccentricity mass is located at disc 2 on node 11. The eccentric moment is calculated to be $m_e = 0.9 \times 10^{-4}$ kg·m, where the eccentric mass, $m_e$, is 3 g and the eccentric radius, $e$, is 30 mm. Rub-impacts occur at disc 2 in the horizontal direction. The clearance of the rub point, $\delta_0$, is set as 70 μm. The friction coefficient, $f_r$, is 0.1 N/(m/s) and the limiter stiffness, $k_r$, is $0.5 \times 10^5$ N/m. Figs 5(a) to (c) show that the rotor vibrations are periodic, which is similar to the experimental result in Fig. 5(d).

3.2 Case 2

The typical vibrations at a rotating speed of 70 Hz are shown in Fig. 6. The eccentricity mass is at disc 2. The eccentric moment is $m_e = 0.6 \times 10^{-4}$ kg·m, where $m_e = 2$ g and $e = 30$ mm. The rub-impact occurs at node 11 in a horizontal direction. The rub clearance and limiter stiffness are the same as Case 1. Different from case 1, the rub friction, $f_r$, is set as a higher value of 0.3 N/(m/s). Figs 6(a) to (c) show that the rotor vibrations are multi-periodic with obvious double harmonics, which is in agreement with the experimental results in Fig. 6(d) as well.
Fig. 6 Transverse vibrations of the dual-disc rotor system with rub-impact occurring on a limiter at the rotating speed of 70 Hz: (a) displacements of node 11; (b) amplitude spectra; (c) shaft orbit of node 11; (d) measured shaft orbit near node 11 from experiment

3.3 Case 3

In this case, the rotating speed is set at 82 Hz and \( m_e = 0.6 \times 10^{-4} \text{ kg m} \), where \( m_e = 2 \text{ g} \) and \( e = 30 \text{ mm} \). The rub-clearance, rod stiffness, and rubbing friction are the same as Case 1. The rotor vibrations of this case are shown in Fig. 7.

Figs 7(a) to (c) show that the rotor vibrations are quasi-periodic motions with super-harmonics, which is regarded as chaos by some researchers, as stated in reference [11]. The FE model motions seen in Figs 7(a) to (c) are in agreement with the experimental measurement qualitatively, shown in the Fig. 7(d).

Additionally, in Figs 5(b), 6(b), and 7(b), it can be seen that super-harmonics occurring in the \( x \) direction are much stronger than that in the \( y \) direction. This is due to the rub-impact being in the \( x \) direction. For shaft orbit, the trend from periodic to chaos indicates the influence of the increasing rotating frequency.

4 TIME-FREQUENCY ANALYSES OF MULTI-PERIODICAL MOTIONS OF THE ROTOR SYSTEM BASED ON HHT

Empirical mode decomposition (EMD) and Hilbert envelope analysis are applied to transverse vibrations of the FE model rotor system to describe the possible instantaneous processes of the rub-impacts for the above three typical cases.

With EMD, a non-linear and non-stationary signal is decomposed into a series of zero-mean amplitude-modulation frequency-modulation (AM-FM) components, called intrinsic mode functions (IMFs), which represent the characteristic time scale of the observation. The IMFs satisfy the following requirements:

1. The number of extremes and the number of zero crossings in the IMF must either be equal or differ at most by one.
2. At any point, the mean value of the envelopes defined by the local maxima and local minima must be zero.

The process to find the IMFs of a signal \( x(t) \) comprises the following steps:

1. Find the positions and amplitudes of all local maxima and minima in the input signal, \( x(t) \), then create an upper envelope by cubic spline interpolation of the local maxima, and a lower envelope by cubic spline interpolation of the local minima. Calculate the mean of the upper and lower envelopes, which is defined as \( m_1(t) \). Subtract the envelope mean from the original input signal

\[
h_1(t) = x(t) - m_1(t) \quad (14)
\]

Check whether \( h_1(t) \) meets the requirements to be an IMF. If not, treat \( h_1(t) \) as new data and repeat the previous process. Then set

\[
h_{11}(t) = h_1(t) - m_{11}(t) \quad (15)
\]

where \( m_{11}(t) \) is the mean of the upper and lower envelopes of obtained \( h_1(t) \).

Repeat this sifting procedure \( k \) times until \( h_{1k}(t) \) is an IMF; this is designated as the first IMF

\[
c_1(t) = h_{1k}(t) \quad (16)
\]

2. Subtract \( c_1(t) \) from the input signal and define the remainder, \( r_1(t) \), as the first residue

\[
r_1(t) = x(t) - c_1(t) \quad (17)
\]

Since \( r_1(t) \) still contains information related to longer period components, it is treated as a new data stream and the above-described sifting process is repeated.

This procedure can be repeated \( j \) times to generate \( j \) residues, \( r_j(t) \), resulting in

\[
r_2(t) = r_1(t) - c_2(t), \ldots, r_j(t) = r_{j-1}(t) - c_j(t) \quad (18)
\]
The sifting process is stopped when either of two criteria are met:
(a) the component, \( c_n(t) \), or the residue, \( r_n(t) \), becomes small enough to be considered inconsequential;
(b) the residue, \( r_n(t) \), becomes a monotonic function from which an IMF cannot be extracted. Thus, we finally obtain
\[
x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t)
\]
(19)

By the above steps, the original signal can now be represented as the sum of a set of intrinsic mode functions plus a residue.

Now apply the Hilbert transform to all IMFs
\[
H[c_j(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{c_j(\tau)}{t-\tau} d\tau
\]
(20)

After the Hilbert transform, \( H[c_j(t)] \) and \( c_j(t) \) together form a complex signal. So the envelope of every IMF, \( c_j(t) \), is given by
\[
a_j(t) = \sqrt{c_j^2(t) + (H[c_j(t)])^2}
\]
(21)

the phase functions are
\[
\Phi_j(t) = \arctan \frac{H[c_j(t)]}{c_j(t)}
\]
(22)

and instantaneous frequencies are obtained as
\[
\omega_j(t) = \frac{d\Phi_j(t)}{dt}
\]
(23)

Here the transverse vibration displacements of the FE model rotor system with rub-impact at fixed limiter are analysed with HHT technique. As shown in section 3, the multi-periodical motions of three typical cases are different from each other. The following Figs 8, 9 and 10 are the calculated EMDs and instantaneous frequencies of the node 11 displacements in the horizontal direction, i.e. x11.

4.1 Case 1: rub-impact occurs at node 11 with rotating speed of 57.5 Hz

In Fig. 8(a), the first component, \( c_1 \), is the high frequency component and represents main periodic motions. The corresponding relatively smooth instantaneous frequency, \( \omega_1 \), coincides with this. The non-smooth instantaneous frequency of \( \omega_2 \) in Fig. 8(b) shows that a slight fluctuation of the main rotating period exists. The slight changes of instantaneous frequency, \( \omega_2 \), indicate the initiation of the rub-impact.

4.2 Case 2: rub-impact occurs at node 11 with rotating speed of 70 Hz

In this case, the rotor transverse vibrations show double harmonics. The first and second IMFs of \( c_1 \) and \( c_2 \), together their corresponding instantaneous frequencies of \( \omega_1 \) and \( \omega_2 \), indicate the above double periodical motions clearly, as shown in Fig. 9. This coincides with the results in Fig. 6. It can be seen from Fig. 9 that double harmonics is characteristic when rotational speed increases in case of the same initial rubbing clearance.

4.3 Case 3: rub-impact occurs at node 11 with rotating speed of 82 Hz

In this case, the first two IMFs are important periodical components of the rotor vibrations with serve rub-impacts. From the first three instantaneous frequencies of \( \omega_1 \), \( \omega_2 \), and \( \omega_3 \), it can be seen that the
periodical motions of the rotor is complex, which is quite different from the previous two cases.

The IMFs and instantaneous frequencies of these three typical cases are indicators of the non-stationary behaviour of the rotor system with multi-periodical vibrations due to rub-impacts. The motion patterns change as shown in the IMF plots and the instantaneous frequencies of signals are also fluctuating due to rub-impacts.

5 CONCLUSIONS

This paper studies the rub-impact problem of the rotor system with fixed limiters. The results show that the vibration of the system is with characteristic periodic motions, and such periodic motions are more prone to occur in the rotor system with a fixed limiter than in the rotor system with common rubbings. The developed FE model not only has a good accuracy when comparing the results from experiments, but also has the ability to analyse multi-disc rotors efficiently and reflect the non-linear components of rub-impact in a relatively clear way.

According to previous work, rotating speed and rotor unbalance (eccentric moment) are the factors which affect rotor rubbing vibration seriously. The current research shows that the rubbing clearance, the stiffness of the rubbing staff, and the rubbing friction coefficient also affect the extent of rubbing and the rubbing vibration mode. In particular, the stiffness of the rubbing staff is found to affect the periodic patterns of rotor vibrations, while a greater friction coefficient leads to new double periodical motions. These different patterns of multi-periodic motions were also observed in corresponding experiments, and the experimental results have good agreement with those in simulation.

HHT analysis of the rotor system with rub-impact at a fixed limiter also reveals that, even with different rub-impact severity and/or different vibration patterns, there exists both non-stationary time-frequency
characteristics and relatively distinctive periodicities in transverse vibration as shown in IMFs and instantaneous frequencies.

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APPENDIX

Notation

A the section area of a beam element
B local nodes of a beam element, as a subscript: relative to node A or B
C total viscous damping matrix
E the Young’s modulus of a beam
f_i friction coefficient at rub-impact point
F_{g,r}, F,u, F_{rub} total gravity force vector, unbalance force vector, and rub-impact force vector of the rotor system
F_{e} gravity vector of an element
F_{g,r}, F_{u} unbalance force vector of an element
F,r, F_N radical and normal rub-impact force components
F_{rub,i} rub-impact force vector at a node i
F_x, F_y rub-impact force components at a given point in x and y directions
G^e gyroscopic moment matrix of a beam element
the cross inertial moment of beam element
axial stiffness coefficient of the rubbing limiter
stiffness and damping coefficients of a journal bearing
beam element length
unbalance mass and eccentricity distance
total mass, damping and stiffness matrices of the FE model rotor system
element mass matrix and element stiffness matrix
element translating inertial matrix and element rotating inertial matrix
radius of a beam element
nodal displacement vector of the rotor system, its dimension is $4n \times 1$
general displacement vector of a beam element for shaft node displacements in the $x$ and $y$ directions respectively
(the global coordinate system, where the $z$ axis is coincident with the rotational axis of the rotor)
two coefficients of the total viscous damping matrix
initial clearance between the shaft or disk and the fixed limiter
rotations about the $x$ and $y$ axes respectively ($i = 1, \ldots, n$ or $i = A$ or $B$)
density of a beam element
first and second natural frequency of the rotor system
rotating speed of the shaft